RESEARCH STATEMENT

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The focus of my research is at the intersection of algebraic geometry on one side and differential geometry or symplectic topology, on the other side. The main theme is to *understand how the complex structures on a given smooth manifold are related to the underlying* $\mathcal{C}^\infty-$*structure.* A common idea behind my work is the interplay between these structures. My contributions to this question are the following:

• I used results in algebraic topology to construct examples of Kähler manifolds regarding the smooth invariance of the Kodaira dimension.

• I approached questions in Kähler geometry using the theory of minimal models, and the theory of rationally connected manifolds. In this direction, I have studied the existence and the properties of Kähler metrics of positive total scalar curvature.

• I studied the asymptotic behavior of the Gromov-Witten-Welschinger invariants of various real algebraic 4-manifolds, in the presence of a small number of real constraints.

• I studied the interpretation in the algebraic setting of the rational blowdown, a construction typically restricted to the class of symplectic 4-manifolds. I obtained results concerning the existence of interesting examples of complex surfaces of Kähler type.

In the next section I will describe the results proved in my thesis, followed by a description of the results recently obtained. I will conclude with a section about my future projects.

1. Past research

The results presented in this section are part of my PhD thesis, and the content of [R06] and [R05].

1.1. Smooth topology and Kodaira dimension.

The question addressed is how the Kodaira dimension of an integrable almost complex structures on a given smooth manifold is related to the underlying $\mathcal{C}^\infty-$structure. I provided examples of Kähler manifolds exhibiting the following behavior:

Theorem 1.1. [R06] *For any* $d, d' \in \{-\infty, 0, 1, 2, 3\}$, $d \neq d'$ *with the exception of* $(-\infty, 0)$ *and* $(0, 3)$, *there exist infinitely many pairs of compact, diffeomorphic Kähler three dimensional manifolds* (M, M'), *having the same Chern numbers, but with Kodaira dimensions* $Kod(M) = d$ *and* $Kod(M') = d'$, *respectively.*

For complex surfaces the Kodaira dimension is [FQ94] a smooth invariant. As an immediate consequence of the above theorem, I settled the question of the smooth invariance of the Kodaira dimension in the case Kähler threefolds:

Corollary 1.2. [R06] *For Kähler threefolds, the Kodaira dimension is not a smooth invariant.*

The examples I found provide answers to questions about the deformation type, too. Recall that deformation equivalent manifolds are orientedly diffeomorphic $\mathcal{C}^\infty-$manifolds.

The converse is known as the $DIF \Rightarrow DEF$ problem. There are known, complicated, counterexamples to this problem in complex dimension 2 or more. Since an essential feature of the Kodaira dimension is its invariance under small deformations, Theorem 1.1 yields simple counterexamples in complex dimension three to the $DIF \Rightarrow DEF$ problem.

In the same vein, I extended Ruan's results [Ru94], who found examples of diffeomorphic, non deformation equivalent Kähler three-folds of negative Kodaira dimension:

Theorem 1.3. [R06] *For any $d \in \{-\infty, 0, 1, 2, 3\}$, there exist infinitely many pairs (M, M') of compact, diffeomorphic, non-deformation equivalent Kähler manifolds of dimension three, with the same Chern numbers, and with Kodaira dimensions satisfying $Kod(M) = Kod(M') = d$.*

1.2. **On the Total Scalar Curvature of Kähler threefolds.** One of the weakest positivity conditions to impose on a metric of a Riemannian manifold is the positivity of the total scalar curvature. However, this condition becomes much stronger when the metric is required to be Kähler. More precisely, for compact, complex manifolds the existence of a Kähler metric of positive total scalar curvature forces the Kodaira dimension to be negative. It is an open problem whether this condition is sufficient or not.

I have studied the *existence* and the *properties* of Kähler metrics of positive total scalar curvature for manifolds of negative Kodaira dimension from the algebraic perspective. In this context, the questions to be explored became the existence and the properties of ample line bundles H satisfying $K_X \cdot H^{n-1} < 0$, where K_X denotes the canonical bundle of X. The best results I obtained were restricted to a class of manifolds intensely studied in the recent years. It was the class of *rationally connected manifolds* [KMM92], i.e. the class of those smooth, projective manifolds whose points can be joined by rational curves. It should be pointed out that in this case, looking for Kähler metrics or for ample line bundles are equivalent tasks.

Theorem 1.4. [R05] *For rationally connected manifolds, the existence of Kähler metrics of positive total scalar curvature is an open property under small deformations.*

The existence question is more delicate, and it can be connected to deep results of Miyaoka and Mori. The answer is known to be *yes* in complex dimensions 1 and 2, using the known classification results.

The focus is exclusively on the case of projective threefolds. We will say that for a given projective variety with $\mathbb{Q}-$Cartier canonical divisor K_X, the property $\mathcal{P}_X$ holds true if there exists an ample line bundle H on X such that $K_X \cdot H^2 < 0$.

As in the case of complex surfaces, the idea to approach this problem is via the minimal models theory[1]. This requires working with singular varieties.

Let X be a locally $\mathbb{Q}-$factorial threefold of Kodaira dimension $\mathrm{Kod}(X) = -\infty$, with at most terminal singularities. In this case, there exists a locally $\mathbb{Q}-$factorial variety X_{min}, with at most terminal singularities, and a birational map $\Phi : X \dashrightarrow X_{min}$, where X_{min} is a del Pezzo fibration, a conic bundle or a $\mathbb{Q}-$Fano variety. It is easy to see that $\mathcal{P}_{X_{min}}$ holds true, but a difficult task is to prove that the property $\mathcal{P}$ can be lifted to X.

As a first step for a better understanding of the problem, I proved the following:

Proposition 1.5. *Let $p : X' \to X$ be a resolution of singularities of a projective, locally $\mathbb{Q}-$factorial variety X of dimension three, with terminal singularities, which is an isomorphism outside the singular locus of X. Then $\mathcal{P}_{X'}$ holds true if and only if $\mathcal{P}_X$ holds true.*

[1]For definitions and main results in the minimal model program we refer to [KM98]

This shows that it will be enough to prove that $\mathcal{P}$ is a birational property. We can use now the weak factorization theorem, which says that any birational map between smooth (projective) manifolds can be decomposed into a finite sequence of blow-ups and blow-downs with nonsingular centers, of (projective) manifolds. I proved the following:

Theorem 1.6. [R05] *For any smooth, projective threefolds, the property $\mathcal{P}$ has the following birational behavior:*

1) *Let Y be the blow-up of X at a point. Then $\mathcal{P}_X$ holds true if and only if $\mathcal{P}_Y$ holds true.*
2) *Let Y be the blow-up of X along smooth curve. If $\mathcal{P}_X$ holds true then $\mathcal{P}_Y$ holds true.*
3) *Let Y be the blow-up of X along a smooth curve C with $K_X \cdot C < 0$. If $\mathcal{P}_Y$ holds true then $\mathcal{P}_X$ holds true.*

Remark 1.7. *In the case of blowing up along curves, more is true. When $K_X \cdot C = 0$, and $\mathcal{P}_Y$ holds true, then $\mathcal{P}_X$ holds true, with one possible exception, $C \simeq \mathbb{P}_1$ and $N_{C/X} \cong \mathcal{O}_{\mathbb{P}_1}(-1) \oplus \mathcal{O}_{\mathbb{P}_1}(-1)$.*

I proved [R05] that for rationally connected threefolds, the only potential obstructions towards the existence of Kähler metrics of total scalar curvature are essentially the ones observed in Remark 1.7.

Theorem 1.8. [R05] *Suppose that for any rationally connected threefold X and Y its blow-up along a smooth rational curve C with $N_{C/X} \cong \mathcal{O}_{\mathbb{P}_1}(-1) \oplus \mathcal{O}_{\mathbb{P}_1}(-1)$, the property $\mathcal{P}_X$ holds true provided that $\mathcal{P}_Y$ holds true. Then for any smooth, rationally connected threefold, property $\mathcal{P}$ holds true.*

2. Present work

I am currently working on two different areas of study. The first one is the real enumerative geometry, where I try to understand and contribute to the theory of Gromov-Witten-Welschinger invariants. The second area is the rational blow-down and its application to the differential geometry of 4-manifolds.

2.1. **Real enumerative geometry.**

As a postdoc at IRMA, Université Louis Pasteur, Strasbourg, under the guidance of V. Kharlamov and J.-Y. Welschinger, I became interested in the rapidly developing field of real enumerative geometry. This is my current theme of research.

Let (X, ω, ϕ) be a compact differentiable $4-$manifold endowed symplectic form ω and an involution ϕ on X satisfying $\phi^*\omega = -\omega$, and $d \in H_2(X, \mathbb{Z})$ a homology class satisfying $c_1(X)d > 0$ and $\phi_* d = -d$. We assume the fixed locus of ϕ is connected and we fix collections of generic r real and m pairs of conjugated imaginary generic points of X, where $r + 2m = c_1(X)d - 1$. In his seminal paper [W05], Welschinger defined invariants under deformation of real symplectic 4-manifolds, a kind of real analogs of Gromov-Witten invariants. These invariants count with signs the number of real rational $J-$holomorphic curves which realize a given homology class and pass through a given real configuration of generic points, and therefore providing lower bounds for the number of such curves. We denote by $W_r^d(X)$ the associated Gromov-Witten-Welschinger invariant.

Unlike the Gromov-Witten theory which is well established, the study of the Gromov-Witten-Welschinger invariants is still in its infancy. There are many interesting questions stemming from the analogy with the Gromov-Witten invariants to ask. Until now, I have approached the following:

A: Asymptotic behavior of the Gromov-Witten-Welschinger invariants. An interesting direction to explore was proposed in [IKS04] and consists in understanding the asymptotic behavior of the Gromov-Witten-Welschinger invariants, and its relation with the asymptotic behavior of the Gromov-Witten invariants. With methods specific to the symplectic field theory, I have studied the asymptotic behavior of Gromov-Witten-Welschinger invariants for small values of r. Complementing the results in [IKS04], I obtained the following:

Theorem 2.1. [RW08] *Let r, m, d be positive integers such that $r + 2m = 3d - 1$. For $r = 0$ or $r = 1$ we have:*

$$\lim_{d\to\infty} \frac{\log |W_r^d(\mathbb{CP}^2)|}{d \log d} = \frac{3}{2} = \frac{1}{2} \lim_{d\to\infty} \frac{\log GW_d(\mathbb{CP}^2)}{d \log d}.$$

In the case of the quadric ellipsoid, i.e. $X = \mathbb{CP}^1 \times \mathbb{CP}^1$ and $Fix(\phi) = S^2$, in a joint work with J.-Y. Welschinger [RW08] we obtained the following:

Theorem 2.2. [RW08] *Let (X, ω, ϕ) be a real symplectic 4-manifold, symplectomorphic to the quadric ellipsoid, d a positive integer, $r \in \{1, 3\}$, and h the class of a real hyperplane section of bidegree $(1, 1)$. We have:*

$$\lim_{d\to\infty} \frac{\log |W_r^d(X)|}{d \log d} = 2 = \frac{1}{2} \lim_{d\to\infty} \frac{\log GW_d(X)}{d \log d}.$$

B: Relative Gromov-Witten-Welschinger invariants. In analogy with the Gromov-Witten theory, I am trying to fill a big gap, by developing a theory of *relative Gromov-Witten-Welschinger invariants*. In a joint work [RSo08] with J. Solomon, we were able to define relative Gromov-Witten-Welschinger invariants relative to codimension 2 real symplectic submanifolds of symplectic 4-manifolds endowed with a compatible real structure.

From the point of view of [S06], the Gromov-Witten-Welschinger invariants can be seen as counting with signs the number of $J-$holomorphic disks with lagrangian boundary conditions subject to point constraints, where the lagrangian is the fixed locus of a real structure. It is the same point of view we adopted in [RSo08], where we define *relative open Gromov-Witten invariants*.

To describe our results, let (X, ω, ϕ) be a real symplectic manifold with dim $X = 4$, $V \subset X$ a real symplectic submanifold of codimension 2, and $d \in H_2(X, \mathbb{Z})$ such that $c_1(X)d > 0$. Let $\mathbb{R}X = Fix(\phi)$ and $\mathbb{R}V = Fix(\phi_{|V}) \subset \mathbb{R}X$, which for convenience are assumed to be non-empty and connected. We denote by $\mathbf{a} = (a_1, a_2, \ldots, a_s)$, and $\mathbf{A} = (A_1, A_2, \ldots, A_t)$ the multiplicity vectors at prescribed boundary and interior points, respectively. Similarly, we denote by $\mathbf{b} = (b_1, a_2, \ldots, b_r)$, and $\mathbf{B} = (B_1, B_2, \ldots, B_u)$, the multiplicity vectors at un-prescribed boundary and interior points, respectively. These vectors record the contact points of real $J-$holomorphic curves realizing the class d with V. We defined relative open Gromov-Witten invariants $OGW_k^l(X, V, \phi, \mathbf{a}, \mathbf{A}, \mathbf{b}, \mathbf{B})$ with respect to V, counting real $J-$holomorphic curves with appropriate signs, passing through a generic collection of k real points and l pairs of generic imaginary points, subject to contact conditions with V described by the vectors $\mathbf{a}, \mathbf{A}, \mathbf{b}, \mathbf{B}$.

Theorem 2.3. [RSo08] *Let (X, ω, ϕ) be a real symplectic manifold with dim $X = 4$, $V \subset X$ a real symplectic submanifold of codimension 2, and $d \in H_2(X, \mathbb{Z})$ such that $\phi_* d = -d$. Assume $\mathbb{R}X$ is relatively $Pin^\pm$ and fix relative $Pin^\pm$ structures on $(X, \mathbb{R}X)$ and on $(V, \mathbb{R}V)$.*

Let $k, l, \mathbf{a}, \mathbf{A}, \mathbf{b}, \mathbf{B}$ as above, satisfying

$$\mu(d) - 1 = k + 2l + \sum_{i=1}^{s} a_i + \sum_{i=1}^{r} b_i + 2\sum_{i=1}^{t} A_i + \sum_{i=1}^{u} 2B_i - r - 2u,$$

where μ denotes the Maslov index. If $\mathbb{R}X$ is orientable, we fix its orientation. If not, assume that $w_1(d) = k + s + 1 \mod 2$ and all a_i and b_i are odd. Then the integers $OGW_k^l(X, V, \phi, \mathbf{a}, \mathbf{A}, \mathbf{b}, \mathbf{B})$ neither depend on the choice of the almost complex structure, nor on the choice of the generic constraints.

We should remark that a similar result was previously discovered in the context of tropical relative Welschinger invariants [IKS06] by Itenberg, Kharlamov and Shustin. Our result extends the relative invariants [W06].

2.2. **The algebraic rational blowdown.** For a long the time, the only known rich source of symplectic manifold was given by the class of Kähler manifolds. Two important techniques of constructing new symplectic manifolds, not necessarily of Kähler type, are Gompf's normal connected sum and the rational blowdown construction of Fintushel and Stern. The rational blowdown is a surgery procedure, specific to the $4-$dimensional realm which amounts to removing a neighborhood of a linear chain of embedded spheres and gluing in a rational ball.

Any symplectic manifold admits (compatible) almost complex structures. It is known that when starting with Kähler surfaces, the symplectic manifolds obtained by the above procedures do not always carry integrable almost complex structures. The question to address is *when the rational blowdown can actually be performed in the algebraic setting.* The main focus is on interesting examples with the purpose of understanding the existence or non-existence of "good" metrics on 4-manifolds. I will describe next the results I obtained in this direction.

In a joint work [RS06] with Ioana Şuvaina, we provide a sufficient criterion to achieve complex structures on the rational blowing down:

Theorem 2.4. [RS06] *Let G be a finite group acting with only isolated fixed points on a smooth, compact, complex surface S with $H^2(S, \Theta_S) = 0$. If the singularities of S/G are of class T, then the rational blowing down $\widetilde{S}$ of the minimal resolution of S/G admits complex structures. Moreover, as a smooth $4-$manifold, $\widetilde{S}$ is orientedly diffeomorphic to a 1-parameter $\mathbb{Q}-$Gorenstein smoothing of S/G.*

For convenience, we recall that the singularities of class T are quotient singularities admitting 1-parameter $\mathbb{Q}-$Gorenstein smoothing. The exceptional divisor of the minimal resolution of these singularities is a linear chain of rational curves on which the rational blowing down can be performed.

As an application, we find a complex structure on an example constructed by Gompf. To briefly recall Gompf's example, we start with a simply connected, relatively minimal elliptic complex surface, with no multiple fibers, and with Euler characteristic $c_2 = 48$. Let $W_{4,n}$ be it's rational blowdown of n sections of self-intersection -4, $n = 1, \ldots, 9$. We were able to show that $W_{4,8}$ admits an integrable complex structure, whose existence was left as an open problem in [G95].

Theorem 2.5. [RS06] *The $4-$manifold $W_{4,8}$ admits a compatible integrable complex structure.*

The emphasis is on the method employed. It is based on the interpretation [M01] of the rational blowing down in algebraic setting as the 1-parameter $\mathbb{Q}-$Gorenstein smoothing of surface singularities of class T.

Since [RS06] has been posted, the $\mathbb{Q}-$Gorenstein smoothing technique has been successfully employed [LP07], [PPS07] to construct examples of simply connected surfaces of general type with $p_g = 0$, and $K^2 = 2, 3$, or 5. In [RS07], we explore the properties of these examples, and we find:

Theorem 2.6. [RS07] *There exist simply connected surfaces of general type with* $p_g = 0,\ K^2 = 2, 3, 4$ *or* 5 *with ample canonical bundle.*

This allows us to prove an interesting result regarding the existence of Einstein metrics on $\mathbb{CP}^2\#k\overline{\mathbb{CP}^2}$, for $k = 5, 6, 7, 8$.

Theorem 2.7. [RS07] *Each of the 4-manifolds* $\mathbb{CP}^2\#k\overline{\mathbb{CP}^2}$, *for* $k = 5, 6, 7, 8$ *admits a differential structure which has an Einstein metric of scalar curvature* $s > 0$, *a differential structure which has an Einstein metric with* $s < 0$ *and infinitely many non-diffeomorphic differential structures which do not admit Einstein metrics.*

As a corollary, in higher dimensions we have:

Corollary 2.8. [RS07] *Let* $N_l = \mathbb{CP}^2\#(l+4)\overline{\mathbb{CP}^2}$, *where* $l \in \{1, 2, 3, 4\}$. *Then the manifold* N *obtained by taking the* $k-$*fold products,* $k \geq 2$, *of arbitrary* N_1, N_2, N_3 *or* N_4, *admits two Einstein metrics* g_1, g_2 *such that their scalar curvatures are* $s_{g_1} = -1, s_{g_2} = +1$. *Moreover, these metrics are Kähler-Einstein with respect to two distinct complex structures* J_1, J_2.

The above examples proved to be very useful answering questions related to the evolution equation associated to the Ricci flow. I will describe next the results [IRS08] I have recently obtained in this direction in a joint work with Ishida and Şuvaina.

I would like to focus our attention on closed oriented Riemannian manifolds of dimension four and on a flow which preserves the volume of the manifold M [H99]:

$$\frac{\partial}{\partial t}g(t) = -2Ric_{g(t)} + \frac{1}{2}\Big(\frac{\int_M s_{g(t)}d\mu_{g(t)}}{\int_M d\mu_{g(t)}}\Big)g(t),$$

where $Ric_{g(t)}, s_g(t)$ are the Ricci, scalar curvature of the evolving Riemannian metric $g(t)$ and $d\mu_{g(t)}$ is the volume measure with respect to $g(t)$. We are interested in solutions which are defined for all time $t \in [0, \infty)$ and for which the sectional curvature tensor $Rm_{g(t)}$ of $g(t)$ is uniformly bounded $\sup_{M\times[0,\infty)} |Rm_{g(t)}| < \infty$. Such solutions were first introduced and studied by Hamilton [H99] and are called *non-singular solutions*. While studying this situation Fang, Zhang and Zhang [FZZ08] proved that the existence of such solutions is topologically obstructed. Using the estimates on the curvature components yielded by non-trivial solutions of the Seiberg-Witten equations Ishida [I08] found new obstructions to the existence of non-singular solutions depending on the diffeotype. His obstructions are on manifolds with $b^+(M) \geq 2$. In joint work with Ishida and Şuvaina, we were able to extend his result to manifolds with $b^+ = 1$.

Theorem 2.9. [IRS08] *Let* X *be a closed oriented smooth 4-manifold with* $b^+(X) \geq 1$, $2\chi(X) + 3\tau(X) > 0$ *and assume that* X *has a non-trivial Seiberg-Witten invariant. Let* N *be a closed oriented smooth 4-manifold with* $b_1(N) = b^+(N) = 0$. *Then, there do not exist non-singular solutions to the normalized Ricci flow on* $M := X\#N$ *if*

$$b_2(N) > \frac{1}{3}\Big(2\chi(X) + 3\tau(X)\Big).$$

We can use this obstruction to study the manifolds with small topology. We exhibit how the change of the smooth structure translates into distinct differential invariants and different behavior of the solutions of the normalized Ricci flow:

Theorem 2.10. [IRS08] *For $5 \leq \ell \leq 8$, the topological 4-manifold $M := \mathbb{C}P^2 \# \ell \overline{\mathbb{C}P^2}$ satisfies the following properties:*

1. *The canonical smooth structure on M has positive Yamabe invariant and there exists a non-singular solution to the normalized Ricci flow on M.*
2. *M admits a smooth structure of negative Yamabe invariant on which there exist non-singular solutions to the normalized Ricci flow.*
3. *M admits infinitely many distinct smooth structures all of which have negative Yamabe invariant and on which there are no non-singular solutions to the normalized Ricci flow for any initial metric.*

3. Future Projects

My main research projects, which are currently work in progress, are devoted to the study of Welschinger invariants. They are the natural analogues of the well-studied Gromov-Witten invariants. However, the theory of Welschinger invariants is still in infancy, and there are many natural questions stemming from the Gromov-Witten theory to be answered.

A: I would like to continue studying the asymptotic behavior of the Gromov-Witten-Welschinger invariants:

1. In a joint work with J.-Y. Welschinger, we started studying an extension of Theorem 2.2 to the case of an arbitrary real symplectic 4-manifold (X, ω, ϕ), where the fixed locus of the real structure ϕ is S^2.
2. The method we employed in the proof of Theorem 2.2 hints that when we fix the number of real constraints r, the following stronger result should be true
$$\lim_{d\to\infty} \frac{\log |W_r^d(X)|}{d \log d} = 2 = \frac{1}{2} \lim_{d\to\infty} \frac{\log GW_d(X)}{d \log d},$$
when the real part of X is S^2. I intend to explore this direction in the near future.

B: Regarding the relative open Gromov-Witten invariants, I would like to continue working on the following projects:

1. In a joint work with J. Solomon, we investigate an extension of Theorem 2.3. We intend to define relative Gromov-Witten-Welschinger invariants of real 6-manifolds (X, ω, ϕ), with respect to a real symplectic submanifold $V \subset X$, with non-empty real part. Since the open Gromov-Witten are defined in [S06] up to dimension 6, this is the last dimension where the relative open Gromov-Witten could be defined.
2. As part of the joint work with J. Solomon, we intend to find a gluing formula for the relative Gromov-Witten-Welschinger invariants, in the spirit of the already existing gluing formula for the relative Gromov-Witten invariants. In particular, we expect to achieve a Caporaso-Harris type formula for these invariants. Such a formula should explain and extend in symplectic setting the results obtained via tropical geometry by Itenberg, Kharlamov and Shustin [IKS06].
3. In real algebraic geometry, a good understanding of the Hurwitz numbers is still missing. With the help of the relative open Gromov-Witten invariants we defined, in a joint work with Solomon, we intend to properly define Hurwitz numbers in genus zero, and to find a recursive formula generating them.

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