HOMOTOPICAL EXCISION, AND HUREWICZ THEOREMS, FOR n-CUBES OF SPACES

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[Received 29 August 1985]

Introduction

The fact that the relative homotopy groups do not satisfy excision makes the computation of absolute homotopy groups difficult in comparison with homology groups. The failure of excision is measured by triad homotopy groups $\pi_n(X; A, B)$, with $n \geqslant 3$ (for $n = 2$, this gives a based set), which fit into an exact sequence

$$\pi_{n+1}(A, A \cap B) \xrightarrow{e} \pi_{n+1}(X, B) \to \pi_{n+1}(X; A, B) \to \pi_n(A, A \cap B) \to \pi_n(X, B) \to \ldots$$

where e is induced by the 'excision' inclusion. That e is an isomorphism in a range of dimensions is shown by the classical Blakers–Massey *triad connectivity theorem*: if A, B, and $A \cap B$ are connected, $\{A, B\}$ is an open cover of X and $(A, A \cap B)$ is p-connected, $(B, A \cap B)$ is q-connected, then the triad $(X; A, B)$ is $(p+q)$-connected. (See, for example, [**11**, p. 211].) Further, if $p, q \geqslant 2$ and $\pi_1(A \cap B) = 0$, the critical group $\pi_{p+q+1}(X; A, B)$ is described in [**2**] as a tensor product of abelian groups $\pi_{p+1}(A, A \cap B) \otimes \pi_{q+1}(B, A \cap B)$.

One of our main results (Theorem 4.2) extends this description of the critical group to the cases where $p, q \geqslant 1$, $\pi_1(A \cap B) \neq 0$. Note that if p or q is 1, then one at least of the groups $\pi_{p+1}(A, A \cap B)$, $\pi_{q+1}(B, A \cap B)$ may be non-abelian, and acts on the other group. In the description of the critical group, the usual tensor product must be replaced by the tensor product $G \otimes H$ defined in [**5, 6**], which involves actions of G on H and H on G. This description of $\pi_{p+q+1}(X; A, B)$ is a special case of a description of the hyper-relative group $\pi_n(X; A_1, \ldots, A_n)$ of a 'connected' excisive $(n+1)$-ad as determined by the lower dimensional information involved in the $(n+1)$-ad; a precise description is given in Theorem 4.1. As another consequence of Theorem 4.1 we obtain an exact sequence for a connected space

$$\pi_2 X \xrightarrow{E^2} \pi_4 S^2 X \xrightarrow{H^2} \pi_1 X \tilde{\wedge} \pi_1 X \xrightarrow{P} [\pi_1 X, \pi_1 X] \longrightarrow 1,$$

where for a group G, the group $G \tilde{\wedge} G$ is obtained from the tensor product $G \otimes G$, with action of G on itself given by conjugation, by factoring by the relations

$$(x \otimes y)(y \otimes x)^{-1} = 1 \quad \text{for all } x, y \in G.$$

Our other results involve a hyper-relative form of homotopical excision. Let

A.M.S. (*1980*) *subject classification*: 55Q30, 55Q15, 55Q52.
Proc. London Math. Soc. (3) 54 (1987) 176–192.

Proceedings of the London Mathematical Society

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Brown and Loday s3-54 (1): 176. (1987)

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