



# Efficient Legendre moment computation for grey level images

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## Abstract

Legendre orthogonal moments have been widely used in the field of image analysis. Because their computation by a direct method is very time expensive, recent efforts have been devoted to the reduction of computational complexity. Nevertheless, the existing algorithms are mainly focused on binary images. We propose here a new fast method for computing the Legendre moments, which is not only suitable for binary images but also for grey level images. We first establish a recurrence formula of one-dimensional (1D) Legendre moments by using the recursive property of Legendre polynomials. As a result, the 1D Legendre moments of order  $p$ ,  $L_p = L_p(0)$ , can be expressed as a linear combination of  $L_{p-1}(1)$  and  $L_{p-2}(0)$ . Based on this relationship, the 1D Legendre moments  $L_p(0)$  can thus be obtained from the arrays of  $L_1(a)$  and  $L_0(a)$ , where  $a$  is an integer number less than  $p$ . To further decrease the computation complexity, an algorithm, in which no multiplication is required, is used to compute these quantities. The method is then extended to the calculation of the two-dimensional Legendre moments  $L_{pq}$ . We show that the proposed method is more efficient than the direct method.

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**Keywords:** Legendre moments; Fast algorithm; Recurrence formula; Grey level images

## 1. Introduction

Since Hu introduced the moment invariants [1], moments and moment functions of image intensity values have been successfully and widely used in the field of image analysis, such as object recognition, object representation, edge detection [2]. Orthogonal moments (e.g. Legendre moment and Zernike moment) can be used to represent an image with the minimum amount of information redundancy [3]. Since the computation of orthogonal moments of a two-dimensional (2D) image by a direct method involves a significant amount of arithmetic operations, some fast algorithms have been developed to reduce the computational complexity. However, the existing methods for fast computation of Legendre moments are mainly focused on binary images [4–6]. Because the moments of a grey level image are also used in many applications, such as texture analysis [7], in this paper we

propose a fast algorithm for computing the Legendre moments for grey level images. The principle is as follows. The recurrence formula of one-dimensional (1D) Legendre moments is firstly established by using the recursive property of Legendre polynomials. The 1D Legendre moment of order  $p$ ,  $L_p = L_p(0)$ , is expressed as a linear combination of  $L_{p-1}(1)$  and  $L_{p-2}(0)$ . Based on this relationship, the 1D Legendre moments  $L_p(0)$  can thus be obtained from the arrays of  $L_1(a)$  and  $L_0(a)$ , where  $a$  is an integer number less than  $p$ . An algorithm based on a systolic array in which no multiplication is required is used to compute these quantities. We then propose an extension of this method to the 2D Legendre moment  $L_{pq}$  computation.

The remainder of this paper is organized as follows. In Section 2, we first describe a new approach for computing the 1D Legendre moments of 1D signal, and then extend this method to the 2D Legendre moment calculation. Section 3 gives the detailed analysis of the computational complexity and some experimental results. Section 4 provides some concluding remarks.

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1 **2. Fast computation of 2D Legendre moments**

3 The  $(p + q)$ th-order Legendre moment of an image with  
intensity function  $f(x, y)$  is defined by

$$L_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^1 \int_{-1}^1 P_p(x) P_q(y) f(x, y) dx dy, \quad (1)$$

5 where  $P_p(x)$  is the  $p$ th-order Legendre polynomial given by

$$P_p(x) = \frac{1}{2^p} \sum_{k=0}^{p/2} (-1)^k \frac{(2p-2k)!}{k!(p-k)!(p-2k)!} x^{p-2k}, \quad (2)$$

$x \in [-1, 1].$

7 For a digital image of size  $N \times N$ , Eq. (1) is usually  
approximated by

$$L_{pq} = \frac{(2p+1)(2q+1)}{(N-1)^2} \sum_{i=1}^N \sum_{j=1}^N P_p(x_i) P_q(y_j) f(x_i, y_j), \quad (3)$$

9 with  $x_i = (2i - N - 1)/(N - 1)$ ,  $y_j = (2j - N - 1)/(N - 1)$ .

11 The Legendre polynomial obeys the following recursive  
relation

$$13 P_{p+1}(x) = \frac{2p+1}{p+1} x P_p(x) - \frac{p}{p+1} P_{p-1}(x), \quad p \geq 1, \quad (4)$$

with  $P_0(x) = 1$ ,  $P_1(x) = x$ .

15 In the following, we present an algorithm for the fast  
calculation of the 2D Legendre moment for grey level images.  
17 For the sake of simplicity, we first consider the computation  
of 1D Legendre moments.

19 For a 1D discrete signal  $f(x_i)$ ,  $1 \leq i \leq N$ , the 1D Legendre  
moment is given by

$$21 L_p = \frac{2p+1}{N-1} \sum_{i=1}^N P_p(x_i) f(x_i). \quad (5)$$

Let us now introduce the following notation:

$$23 L_p(a) = \frac{2p+1}{N-1} \sum_{i=1}^N x_i^a P_p(x_i) f(x_i). \quad (6)$$

25 It can be easily seen that  $L_p = L_p(0)$ . Thus, we turn to the  
fast computation of  $L_p(a)$  in the following:

Substitution of Eq. (4) into Eq. (6) yields

$$27 L_p(a) = \frac{2p+1}{N-1} \sum_{i=1}^N x_i^a \left[ \frac{2p-1}{p} x_i P_{p-1}(x_i) \right. \\ \left. - \frac{p-1}{p} P_{p-2}(x_i) \right] f(x_i)$$

$$= \frac{2p+1}{p} \frac{2p-1}{N-1} \sum_{i=1}^N x_i^{a+1} P_{p-1}(x_i) f(x_i) \\ - \frac{p-1}{p} \frac{2p+1}{2p-3} \frac{2p-3}{N-1} \sum_{i=1}^N x_i^a P_{p-2}(x_i) f(x_i) \quad (7)$$

therefore, we have the following recurrence relation for  $p \geq 2$ : 29

$$L_p(a) = \frac{2p+1}{p} \left[ L_{p-1}(a+1) - \frac{p-1}{2p-3} L_{p-2}(a) \right] \quad (8) \quad 31$$

with

$$L_0(a) = \frac{1}{N-1} \sum_{i=1}^N x_i^a f(x_i) = \frac{1}{N-1} G_N(a), \quad (9) \quad 33$$

$$L_1(a) = \frac{3}{N-1} \sum_{i=1}^N x_i^{a+1} f(x_i) = \frac{3}{N-1} G_N(a+1), \quad (10)$$

$$G_N(a) = \sum_{i=1}^N x_i^a f(x_i). \quad (11) \quad 35$$

The above discussion shows that the 1D Legendre moments  $L_p = L_p(0)$ , for  $p \geq 2$ , can be deduced from the values of  $L_0(a)$  and  $L_1(a)$  where  $a$  is an integer less than  $p$ ,  $L_0(a)$  and  $L_1(a)$  can be obtained by  $G_N(a)$ . The calculation of Eq. (11) needs to distinguish two different cases: odd  $N$  and even  $N$ . 37  
39  
41

(1)  $N = 2L + 1$ :

Since  $x_i = (2i - N - 1)/(N - 1)$ , we deduce from Eq. (11) that 43

$$G_{2L+1}(a) = \sum_{i=1}^{2L+1} \left( \frac{2i-2L-2}{2L} \right)^a f(x_i) \\ = \frac{1}{L^a} \sum_{i=1}^{2L+1} (i-L-1)^a f(x_i), \\ = \begin{cases} \frac{1}{L^a} [-L^a f(x_1) - (L-1)^a f(x_2) \\ - \dots - f(x_L) + f(x_{L+2}) + 2^a f(x_{L+3}) \\ + \dots + L^a f(x_{2L+1})] & a \text{ is odd,} \\ \frac{1}{L^a} [L^a f(x_1) + (L-1)^a f(x_2) \\ + \dots + f(x_L) + f(x_{L+2}) + 2^a f(x_{L+3}) \\ + \dots + L^a f(x_{2L+1})] & a \text{ is even.} \end{cases} \quad (12) \quad 45$$

Eq. (12) can be rewritten as

$$G_{2L+1}(a) = \begin{cases} \frac{1}{L^a} \sum_{i=1}^L i^a g_1(x_i) & a \text{ is odd,} \\ \frac{1}{L^a} \sum_{i=1}^L i^a g_2(x_i) & a \text{ is even} \end{cases} \quad (13) \quad 47$$

1 with

$$g_1(x_i) = f(x_{L+i+1}) - f(x_{L-i+1}), \quad i = 1, 2, \dots, L, \quad (14)$$

$$3 \quad g_2(x_i) = f(x_{L+i+1}) + f(x_{L-i+1}), \quad i = 1, 2, \dots, L. \quad (15)$$

(2)  $N = 2L$ :

5 In this case, Eq. (11) becomes

$$\begin{aligned} G_{2L}(a) &= \sum_{i=1}^{2L} \left( \frac{2i - 2L - 1}{2L - 1} \right)^a f(x_i) \\ &= \frac{1}{(2L - 1)^a} \sum_{i=1}^{2L} (2i - 2L - 1)^a f(x_i), \\ &= \begin{cases} \frac{1}{(2L - 1)^a} \left[ -(2L - 1)^a f(x_1) \right. \\ \quad \left. -(2L - 3)^a f(x_2) - \dots - f(x_L) \right. \\ \quad \left. + f(x_{L+1}) + 3^a f(x_{L+2}) \right. \\ \quad \left. + \dots + (2L - 1)^a f(x_{2L}) \right] & a \text{ is odd,} \\ \frac{1}{(2L - 1)^a} \left[ (2L - 1)^a f(x_1) \right. \\ \quad \left. + (2L - 3)^a f(x_2) + \dots + f(x_L) \right. \\ \quad \left. + f(x_{L+1}) + 3^a f(x_{L+2}) \right. \\ \quad \left. + \dots + (2L - 1)^a f(x_{2L}) \right] & a \text{ is even} \end{cases} \end{aligned} \quad (16)$$

7 or

$$G_{2L}(a) = \begin{cases} \frac{1}{(2L - 1)^a} \sum_{i=1}^L (2i - 1)^a g_3(x_i), & a \text{ is odd,} \\ \frac{1}{(2L - 1)^a} \sum_{i=1}^L (2i - 1)^a g_4(x_i), & a \text{ is even} \end{cases} \quad (17)$$

9 with

$$g_3(x_i) = f(x_{L+i}) - f(x_{L-i+1}), \quad i = 1, 2, \dots, L, \quad (18)$$

$$11 \quad g_4(x_i) = f(x_{L+i}) + f(x_{L-i+1}), \quad i = 1, 2, \dots, L. \quad (19)$$

13 We discuss, in the following two subsections, the way to efficiently calculate  $G_N(a)$  given by Eqs. (13) or (17), according to the different modalities of the 1D signal  $f(x_i)$ .

15 2.1.  $f(x_i) = 1$  for  $i = 1, 2, \dots, N$

In this case, Eqs. (13) and (17) become

$$17 \quad G_{2L+1}(a) = \begin{cases} 0, & a \text{ is odd,} \\ \frac{2}{L^a} \sum_{i=1}^L i^a, & a \text{ is even,} \end{cases} \quad (20)$$

$$G_{2L}(a) = \begin{cases} 0, & a \text{ is odd,} \\ \frac{2}{(2L - 1)^a} \sum_{i=1}^L (2i - 1)^a \\ = \frac{2}{(2L - 1)^a} \\ \quad \times \left( \sum_{i=1}^{2L} i^a - 2^a \sum_{i=1}^L i^a \right), & a \text{ is even.} \end{cases} \quad (21)$$

The above equations show that to obtain the values of  $G_N(a)$ , we only need to calculate the following summation: 19

$$H_M(a) = \sum_{i=1}^M i^a. \quad (22) \quad 21$$

For the computation of Eq. (22), which is just the 1D geometric moment of order  $a$  of a 'binary' signal, we use the formulae proposed by Spiliotis and Mertzios [8] 23

$$\begin{aligned} H_M(1) &= \frac{M(M+1)}{2}, & H_M(2) &= \frac{M(M+1)(2M+1)}{6}, \\ H_M(3) &= \frac{M^2(M+1)^2}{4}, \\ H_M(4) &= \frac{M(M+1)(2M+1)(3M^2+3M+1)}{30}, \end{aligned} \quad (23) \quad 25$$

and for  $a \geq 4$ , the recurrence formula

$$\begin{aligned} &\binom{a+1}{1} H_M(1) + \binom{a+1}{2} H_M(2) \\ &+ \dots + \binom{a+1}{a} H_M(a) \\ &= (M+1)^{a+1} - (M+1), \end{aligned} \quad (24) \quad 27$$

where

$$\binom{i}{j} = \frac{i!}{j!(i-j)!} \quad (29)$$

is a combination number.

2.2.  $f(x_i) \neq f(x_j)$  for some  $i \neq j$  31

Eq. (17) can be written as

$$G_{2L}(a) = \begin{cases} \frac{1}{(2L - 1)^a} \sum_{i=1}^{2L} i^a h_1(x_i), & a \text{ is odd,} \\ \frac{1}{(2L - 1)^a} \sum_{i=1}^{2L} i^a h_2(x_i), & a \text{ is even,} \end{cases} \quad (25) \quad 33$$

where

$$h_1(x_i) = \begin{cases} g_3(x_{(i+1)/2}) & \text{if } i \text{ is odd,} \\ 0 & \text{otherwise} \end{cases} \quad (26) \quad 35$$

$$h_2(x_i) = \begin{cases} g_4(x_{(i+1)/2}) & \text{if } i \text{ is odd,} \\ 0 & \text{otherwise.} \end{cases} \quad (27)$$

Here  $g_3(x_i)$  and  $g_4(x_i)$  are given by Eqs. (18) and (19), respectively. 37

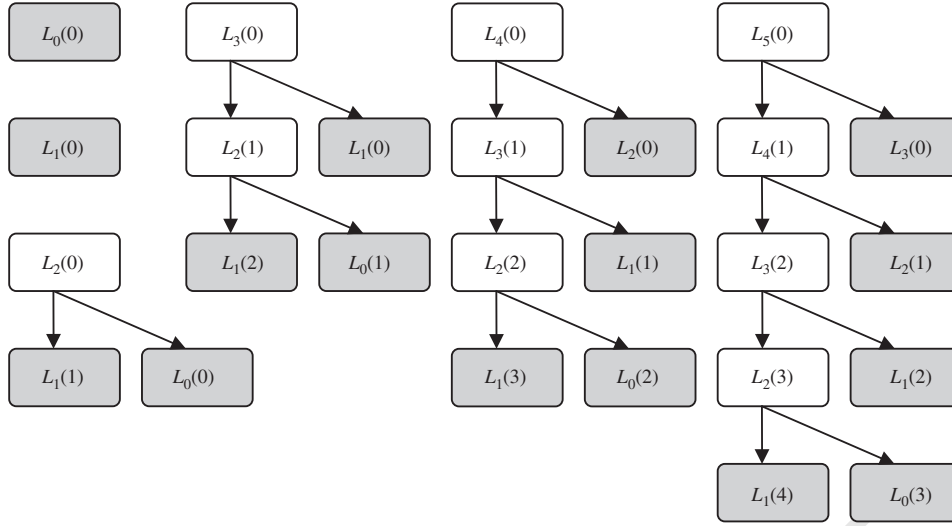


Fig. 1. Computation process of  $L_p(0)$  with  $p$  from 0 to 5. Grey level boxes correspond to already computed coefficients and white boxes to coefficients that will be computed from those which appear in grey level boxes.

```

for i = 1 to N
    computing  $L_0(q)$  ( $0 \leq q \leq M-2$ ) and  $L_1(q)$  ( $0 \leq q \leq M-1$ ) using Eqs. (9) and (10)
    for q = 0 to M
        computing  $Y_{iq}$  for each row  $i$  of the image using Eq. (8)
    endfor
endfor
for p = 0 to M
    computing  $L_0(a)$  ( $0 \leq a \leq M-p-2$ ) and  $L_1(a)$  ( $0 \leq a \leq M-p-1$ ) using Eqs. (9) and
    (10) from pre-calculated  $Y_{iq}$ 
    for q = 0 to M - p
        computing the 2D Legendre moments  $L_{pq}$  using recursive method
    endfor
endfor

```

Fig. 2. Algorithm for computing  $L_{pq}$ .

1 It can be seen from Eqs. (13) and (25) that we need to  
 2 calculate the summation of the form

$$3 \quad S_M(a) = \sum_{i=1}^M i^a g(x_i). \quad (28)$$

4 Note that  $S_M(a)$  is the 1D geometric moment of order  $a$  of  
 5 an arbitrary 1D signal. Since many algorithms are available  
 6 in the literature to speed up the computation of Eq. (28), we  
 7 decided to choose the method proposed by Chan et al. [9].  
 8 Their algorithm is able to efficiently compute the grey level  
 9 image moments. It makes use of a systolic array for computing  
 10 the moments in which no multiplication is required.  
 11 We recently applied such a method to efficiently calculate  
 12 the Zernike moments [10]. For a detailed description of this  
 13 algorithm, please refer to Ref. [10].

14 Thus, the 1D Legendre moments  $L_p(0)$ , for  $0 \leq p \leq M$  ( $M$   
 15 denotes the maximal order we want to calculate), can be ef-

ficiently obtained using the previously presented algorithm.  
 Fig. 1 shows the computation order of  $L_p(0)$  for  $p$  varying  
 from 0 to 5.

Let us now describe the method for fast computation of  
 the 2D Legendre moments  $L_{pq}$ . The double summation in  
 Eq. (3) can be split into the following separate form:

$$\begin{aligned}
 L_{pq} &= \frac{(2p+1)(2q+1)}{(N-1)^2} \sum_{i=1}^N \sum_{j=1}^N P_p(x_i) P_q(y_j) f(x_i, y_j) \\
 &= \frac{2p+1}{N-1} \sum_{i=1}^N P_p(x_i) \left( \frac{2q+1}{N-1} \sum_{j=1}^N P_q(y_j) f(x_i, y_j) \right) \\
 &= \frac{2p+1}{N-1} \sum_{i=1}^N P_p(x_i) Y_{iq}, \quad (29)
 \end{aligned}$$

1 where  $Y_{iq}$  is the  $q$ th-order row moments of row  $i$  given by

$$Y_{iq} = \frac{2q+1}{N-1} \sum_{j=1}^N P_p(y_j) f(x_i, y_j). \quad (30)$$

3 These equations show that the computation of 2D Legendre moments of grey level images can be decomposed into two steps. First, the 1D Legendre moments  $Y_{iq}$ , for  $1 \leq i \leq N$  and  $0 \leq q \leq M$ , are computed by using the algorithm described in Sections 2.1 and 2.2, according to the different image modalities of  $f(x_i, y_j)$ . Then, the row moments  $Y_{iq}$  are applied to compute the 2D Legendre moments  $L_{pq}$ . Thus, after the first step, the 2D Legendre moments  $L_{pq}$  can be calculated as 1D moments by setting the image intensity function  $f(x_i, y_j)$  to the  $Y_{iq}$  previously computed. The algorithm for computing the 2D Legendre moments is depicted in Fig. 2. It should be pointed out that such a strategy can also be realized in parallel.

### 3. Computation complexity and experimental results

17 Let the image size be  $N \times N$  pixels, and  $M$  be the maximum order of Legendre moments to calculate. The maximum order  $M$  is usually less than the image size  $N$ .

19 The direct computation of Eq. (3) requires approximately 21  $O(M^2N^2)$  additions and multiplications, respectively.

#### 3.1. Computational complexity of the proposed method for binary images

23 The computation of the geometrical moments up to the order  $M$  of a binary image with  $N \times N$  pixels, requires approximately  $4M$  power calculations,  $2M^2$  multiplications, and  $M^2$  additions (note that these numbers are not dependent on  $N$ ) [8]. The computation of the 2D Legendre moments  $L_{pq}$ , by using the recursive algorithm, needs  $O(NM^3)$  additions and  $O(M^3)$  multiplications. Therefore, the algorithm is very efficient compared with the direct method.

#### 3.2. Computational complexity of the proposed method for grey level images

35 The computational complexity of the method for grey level images takes into account the parity of  $N$ .

(1) For odd values of  $N$ :

37 Let us first consider the number of operations required in the computation of the  $i$ th row moments  $Y_{iq}$  ( $0 \leq q \leq M$ ). Note that the functions  $g_1(x)$  and  $g_2(x)$  defined by Eqs. (14) and (15) are used for odd values of  $N$ . To obtain the values of  $Y_{iq}$ , we must calculate  $G_N(a)$  with Eq. (13) for  $0 \leq a \leq M$ . This step needs only  $(M+1)^2(N/2-1)$  additions. The computation of  $Y_{iq}$  (for  $0 \leq q \leq M$ ) from the pre-calculated  $G_N(a)$ , requires  $M(M-1)/2$  additions and  $2M(M-1)$  multiplications. Therefore, the computation of  $N$  rows of

$Y_{iq}$  (for  $1 \leq i \leq N$ ) needs approximately  $M^2(N^2+N)/2$  additions and  $2M^2N$  multiplications. 47

When all  $Y_{iq}$ , for  $1 \leq i \leq N$  and  $0 \leq q \leq M$ , are obtained, the 2D Legendre moments  $L_{pq}$ , for  $0 \leq p+q \leq M$ , can be calculated in a similar way. The corresponding additions and multiplications are  $M^3N/12 + M^2N$  and  $2M^3/3 + 2M^2$ . 51

In conclusion, the overall computation makes use of  $O(M^2N^2)$  additions and  $O(M^2N)$  multiplications approximately for  $M \leq N$ . 53

(2) For even values of  $N$ : 55

The functions  $h_1(x)$  and  $h_2(x)$ , which are defined by Eqs. (26) and (27), will be used in the computation of  $G_N(a)$ . The only difference between case (2) and case (1) is that Eq. (25) is adopted instead of Eq. (13). The computation of Eq. (25) requires additions twice more than that is needed in Eq. (13). Thus, the total computational complexity is approximately  $O(M^2N^2)$  additions and  $O(M^2N)$  multiplications. 61

#### 3.3. Experimental results and comparison 63

The computational complexities of the proposed algorithm and the direct method are summarized in Table 1. From this table, we can see that the number of additions of the proposed method is in the worst case ( $N$  even and  $M=N$ ) approximately twice of the direct method, but the number of multiplications is smaller, with a ratio of  $3/N$  with regard to the direct method for  $M \leq N$ . For odd values of  $N$ , the number of additions of the proposed method is approximately the same as that of the direct method, but the number of multiplications decreases considerably. Table 2 shows the number of arithmetic operations and the CPU elapsed time of the two methods for some values of  $N$  and  $M$  (the program was implemented in C++ on PIII-M 1G, 384M). In order to further decrease the computation time for even values of  $N$ , the image can be zero-padded to achieve an odd  $N$ . Such a strategy was adopted by Yap et al. in the computation of Krawtchouk moments [11]. Fig. 3(a) shows the original grey level image of size  $256 \times 256$ . The reconstruction results with  $M=40$  of the original image and its zero-padded image of size  $257 \times 257$  are depicted in Fig. 3(b) and (c), respectively. Fig. 3(d) shows the difference image,  $\varepsilon(x, y)$ , between the two reconstructed images where  $\varepsilon(x, y)$  is defined as 85

$$\varepsilon(x, y) = \left| \widehat{f}_1(x, y) - \widehat{f}_2(x, y) \right|, \quad (31) \quad 87$$

where  $\widehat{f}_1(x, y)$  is the reconstructed result of original image and  $\widehat{f}_2(x, y)$  is the reconstructed result of zero-padded image. 89

Note that in both cases, the reconstruction of the image is performed by using the following formula: 91

$$\widehat{f}(x_i, y_j) = \sum_{p=0}^M \sum_{q=0}^p L_{p-q,q} P_{p-q}(x_i) P_q(y_j). \quad (32) \quad 93$$

Table 1  
Comparison of computational complexity for the two methods

		Addition	Multiplication
Direct method		$M^2N^2/2 \approx O(M^2N^2)$	$M^2N^2 \approx O(M^2N^2)$
Our method	$N$ is even	$M^2N^2 + M^3N/6 \approx O(M^2N^2)$	$2M^2N + 2M^3/3 \approx O(M^2N)$
	$N$ is odd	$M^2N^2/2 + M^3N/12 \approx O(M^2N^2)$	$2M^2N + 2M^3/3 \approx O(M^2N)$

Table 2  
Comparison of computation time for the two methods

	Direct method			Our method		
	Addition	Multiplication	Time (ms)	Addition	Multiplication	Time (ms)
$N = 40 M = 40$	1 377 600	2 758 640	70	3 460 320	172 442	60
$N = 41 M = 40$	1 447 340	2 892 130	70	1 762 100	175 644	50
$N = 80 M = 40$	5 510 000	11 024 000	210	12 347 500	300 522	180
$N = 81 M = 40$	5 650 000	11 300 000	210	6 240 000	303 724	140
$N = 256 M = 40$	56 426 500	112 856 000	2113	115 256 000	864 074	1843
$N = 257 M = 40$	56 868 200	113 740 000	2103	57 891 400	867 276	1022

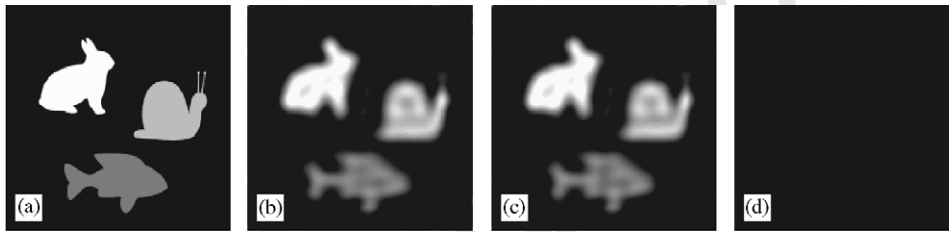


Fig. 3. Comparison of reconstruction results of the image with and without zero-padding ( $M = 40$ ), (a) original image ( $256 \times 256$ ), (b) reconstruction result of original image, (c) reconstruction result of zero-padded image ( $257 \times 257$ ), and (d) error image  $\varepsilon(x, y)$ .

1 It can be seen from Fig. 3(d) that the two reconstructed  
 3 images only have a slight difference, but the computation  
 5 time required in the moment calculation process using the  
 zero-padded strategy, which is 1022 ms (see Table. 2), is  
 much shorter than that of the moment computation based on  
 the original image, which is 1843 ms.

#### 7 4. Conclusion

9 In this paper, a new fast algorithm for computing the 2D  
 Legendre moments of grey level images has been presented.  
 The proposed method has the following advantages:

- 11 (1) The 1D Legendre moments can be obtained by a recur-  
 13 rence relation. Moreover, the initial value used in the  
 iterative method can be calculated with additions only.
- 15 (2) The 2D moment computation can be decomposed into  
 two 1D moment calculations.
- 17 (3) It does not require as many multiplications as the direct  
 method, thus leads to a better efficiency in terms of  
 computational time.
- 19 (4) The algorithm can be implemented in parallel.

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