Conference ANR ProbaGeo at Strasbourg

I.R.M.A. 30, 31 May and 1st June 2011

Organizer : J. Franchi (IRMA)

The purpose is centered on the study of relations between diffusion processes and geometry. More precisely, the following items are addressed : stochastic flows and dynamical systems, path space, heat flow and Ricci lower bounds, relativistic diffusions, hypoelliptic estimates.

Speakers and titles :

- Marc ARNAUDON (Poitiers) : *Stochastic algorithms for computing means of probability measures* ;

- Simone CALOGERO (Granada/Oslo) : *Trend to equilibrium for kinetic Fokker-Planck equations on Riemannian manifolds* ;

- Kolehe COULIBALY (Nancy) : *Stochastic approach to geometric flow, and application to the Ricci flow* ;

- Ana Bela CRUZEIRO (Lisboa) : *Stochastic Lagrangian Navier-Stokes flows* ;

- François DELARUE (Nice) : *Density estimate for a random noise propagating in a chain of differential equations (lower bound)* ;

- Shizan FANG (Dijon) : *Stochastic differential equations with coefficients in Sobolev spaces* ;

- Elton HSU (Northwestern) : *Differentiability of reflecting Brownian motion and its application to path space analysis* ;

- Thierry LÉVY (Paris 6) : *Eigenvalues of Brownian motions on classical groups* ;

- Stéphane MENOZZI (Paris 7) : *Density estimate for a random noise propagating in a chain of differential equations (upper bound)* ;

- Robert PHILIPOWSKI (Bonn) : *Ricci flow, coupling of Brownian motions and Perelman's L-functional* ;

- Camille TARDIF (Strasbourg) : *A study of the dynamics of the stochastic flow of a relativistic diffusion* ;

- Jean-Claude ZAMBRINI (Lisboa) : *Integrable stochastic dynamical systems* ;

Abstracts :

M. ARNAUDON : *Stochastic algorithms for computing means of probability measures*

Consider a probability measure supported by a regular geodesic ball in a manifold. For any p larger than or equal to 1 we define a stochastic algorithm which converges almost surely to the p-mean of the measure. Assuming furthermore that the functional to minimize is regular around the p-mean, we prove that a natural renormalization of the inhomogeneous Markov chain converges in law into an inhomogeneous diffusion process. We give an explicit expression of this process, as well as its local characteristic.

S. CALOGERO : *Trend to equilibrium for kinetic Fokker-Planck eq. on Riemannian manifolds*

A class of linear kinetic Fokker-Planck equations with a non-trivial diffusion matrix and with periodic boundary conditions in the spatial variable is considered. After formulating the problem in a geometric setting, the question of the rate of convergence to equilibrium is studied within the formalism of differential calculus on Riemannian manifolds. Under explicit geometric assumptions on the velocity field, the energy function and the diffusion matrix, it is shown that global regular solutions converge in time to equilibrium with exponential rate. The result is proved by estimating the time derivative of a modified entropy functional. For spatially homogeneous solutions the assumptions of the main theorem reduce to the curvature bound condition for the validity of logarithmic Sobolev inequalities discovered by Bakry and Emery. The result applies to the relativistic Fokker-Planck equation in the low temperature regime, for which exponential trend to equilibrium was previously unknown.

K. COULIBALY : *Stochastic approach to geometric flow, and application to the Ricci flow*

We will talk about some properties of the $g(t)$-Brownian motion. In particular we will discuss the following fact : the family of metrics $g(t)$ evolves under backward Ricci flow if and only if the damped parallel transport becomes equal to the parallel transport. In a second part of this talk we will present a joint work with Arnaudon and Thalmaier, concerning horizontal diffusions in the path space and we will give some applications. We will finish this talk, by giving the Onsager-Machlup functional for the $g(t)$-Brownian motion.

A.B. CRUZEIRO : *Stochastic Lagrangian Navier-Stokes flows*

I will recall how to obtain stochastic Lagrangian flows associated to the Navier-Stokes equations as a solution of a stochastic variational principle (based on recent work with Marc Arnaudon) as well as some of its stability properties. This formulation corresponds to a generalization of V. Arnold's approach to Euler equations and we can write the stochastic analogue of the geodesic equation for Navier-Stokes. Then we can ask how to use the curvature properties of the underlying diffeomorphisms group.

F. DELARUE : *Density estimate for a random noise propagating in a chain of differential equations (lower bound)*

In a row of two talks, given by Stéphane Menozzi and myself, we discuss the way a noise propagates through a finite chain of oscillators over a finite time interval.
The oscillators are modelled by coupled differential equations: the first oscillator only is forced by a random noise, the dynamics of the other ones being purely deterministic. The coupling of the oscillators has some ordering: to reach the oscillator of index i, the noise must go through the oscillators of indices $1, 2, ..., i-1$. Under some natural conditions on the coefficients, the whole system is shown to be of weak Hormander type. The question is then to estimate (from above and from below) its density.
In my talk, I will focus on the lower bound. Stéphane Menozzi will discuss the upper bound. The lower bound is given in two steps: the first one consists in a probabilistic representation of the logarithm of the density as the value function of some stochastic control problem. (This is known as Fleming's transform.) The second one consists in choosing a specific control to bound the value function: when the coefficients of the oscillators are linear, the optimal control is given explicitely; when the system is nonlinear, a possible way consists in choosing a perturbation of the optimal control of the linearized system.

S. FANG : *Stochastic differential equations with coefficients in Sobolev spaces*

This talk will be a survey on joint works with D. Luo, H. Lee and A. Thalmaier, in which we have extended Di Perna-Lions and Ambrosio's methods dealing with ODE to the context of SDE on Euclidian spaces as well as on complete Riemannian manifolds.

E. HSU : *Differentiability of reflecting B. M. and its application to path space analysis*

We will use a pathwise differentiability with respect to the initial position of the reflecting Brownian motion in a euclidean domain as the solution of the Skorokhod equation driven by a Brownian motion. The domain is assumed to have a twice continuously differentiable boundary. The associated discountinuous multiplicative functional can be identified with the one appearing in the Bismut-type probabilistic representation of the logarithmic derivative of the Neumann heat kernel. It is also useful in studying asymptotic behavior of sychronizing coupling of reflecting Brownian motion.

T. LÉVY : *Eigenvalues of Brownian motions on classical groups*

In 1995, Philippe Biane computed the distribution of the eigenvalues of a large unitary matrix taken under the heat kernel measure, using harmonic analysis. This result has been proved since then in different ways, mainly combinatorial, using the relation between the unitary group and the symmetric group, and the geometry of the lattice of non-crossing partitions. We will present a short and self-contained derivation of the limiting moments of the distribution, and an extension of the result to orthogonal and symplectic groups.

S. MENOZZI : *Density estimate for a random noise propagating in a chain of differential equations (upper bound)*

This talk is the sequel of François Delarue's one. We will present the techniques to get an upper bound for the model considered. The key point is to note that for a linear deterministic system perturbed by a non-degenerate Gaussian noise on its first component, the density is Gaussian and can be explicitely controlled. To get the estimate for the non-linear case we then apply a suitable linearization and a perturbative scheme. We follow the parametrix technique of Mc Kean and Singer choosing accurately the Gaussian process. Let us emphasize that to achieve the proof it is necessary to exploit once again the Fleming logarithmic transform to handle the non-linearity through a comparison argument. Another proof of the lower bound can then be derived from the parametrix expansion of the density that directly gives a lower bound on the compact sets of the "control" metric using then a suitable chaining argument. On the other hand, the previous choice of Gaussian process allows as well to discuss the associated martingale problem.

R. PHILIPOWSKI : *Ricci flow, coupling of Brownian motions and Perelman's L-functional*

In this talk I will show that on a manifold whose Riemannian metric evolves under backwards Ricci flow two Brownian motions can be coupled in such a way that their normalized L-distance is a supermartingale. As a corollary, one obtains a new proof and a generalization of a recent result of Peter Topping concerning L-optimal transport. This is joint work with Kazumasa Kuwada.

C. TARDIF : *A study of the dynamics of the stochastic flow of a relativistic diffusion*

Dudley constructed a Lorentz invariant diffusion in the future unit tangent bundle of the Minkowski space $R^{1,d}$, made of the couple of the hyperbolic Brownian motion with its time integral. This diffusion is got by projecting an invariant diffusion (g_t) which lives in the Poincaré group of Lorentzian affine transforms of $R^{1,d}$. We propose here to study the dynamics of the stochastic flow associated to (g_t). In particular we compute the Lyapunov exponents and the stable manifolds in function of the asymptotic variable which describes the Poisson boundary.

J.-C. ZAMBRINI : *Integrable stochastic dynamical systems*

After describing Feynman "space-time" reinterpretation of the uncertainty relations between position and momentum quantum observables, we show a way to give a probabilistic meaning to his result in term of a class of reversible processes. The quest for a similar duality between energy and time suggests to deform stochastically, along those processes, classically integrable systems. Then last entrance and first exit time appear.

Participants

- Jürgen Angst (Rennes, jurgen.angst[at]univ-rennes1.fr) ;
- Marc Arnaudon (Poitiers, Marc.Arnaudon[at]math.univ-poitiers.fr) ; 29-5 $\longrightarrow$ 1-6 ;
- Simone Calogero (Granada, calogero[at]ugr.es) ; 29-5 $\longrightarrow$ 4-6 ;
- Kolehe Coulibaly (Nancy, Kolehe.Coulibaly[at]iecn.u-nancy.fr) ; 29-5 $\longrightarrow$ 1-6 ;
- Ana Bela Cruzeiro (Lisboa, abcruz[at]math.ist.utl.pt) ; 29-5 $\longrightarrow$ 2-6 ;
- Francois Delarue (Nice, Francois.Delarue[at]unice.fr) ; 30-5 $\longrightarrow$ 1-6 ;
- Michel Émery (Strasbourg, michel.emery[at]math.unistra.fr) ;
- Shizan Fang (Dijon, Shizan.Fang[at]u-bourgogne.fr) ; 29-5 $\longrightarrow$ 1-6 ;
- Denis Feyel (Paris, denis.feyel[at]orange.fr) ; 30-5 $\longrightarrow$ 1-6 ;
- Jacques Franchi (Strasbourg, jacques.franchi[at]math.unistra.fr) ;
- Elton P. Hsu (Evanston-Illinois, ehsu[at]math.northwestern.edu) ; 26-5 $\longrightarrow$ 2-6 ;
- Lizhen Ji (Univ. of Michigan, lji[at]umich.edu) ; 30-5 $\longrightarrow$ 1-6 ;
- Nicolas Juillet (Strasbourg, nicolas.juillet[at]math.unistra.fr) ;
- Abdeldjebbar Kandouci (Saïda, kandouci1974[at]yahoo.fr) ;
- Thierry Lévy (Paris, Thierry.Levy[at]upmc.fr) ;
- Stephane Menozzi (Paris, menozzi[at]math.jussieu.fr) ; 30-5 $\longrightarrow$ 1-6 ;
- Shigeyoshi Ogawa (Kyoto, ogawa-s[at]se.ritsumei.ac.jp) ;
- Jean Picard (Clermont-Ferrand, Jean.Picard[at]math.univ-bpclermont.fr) ; 29-5 $\longrightarrow$ 1-6 ;
- Robert Philipowski (Bonn, philipowski[at]iam.uni-bonn.de) ; 29-5 $\longrightarrow$ 1-6 ;
- Abdelkader Saïdi (Strasbourg, saidi[at]math.unistra.fr) ;
- Camille Tardif (Strasbourg, ctardif[at]math.unistra.fr) ;
- Anton Thalmaier (Luxembourg, anton.thalmaier[at]uni.lu) ; 29-5 $\longrightarrow$ 1-6 ;
- Jérémie Unterberger (Nancy, jeremie.unterberger[at]iecn.u-nancy.fr) ;
- Vincent Vigon (Strasbourg, vincent.vigon[at]math.unistra.fr) ;
- Jean-Claude Zambrini (Lisboa, zambrini[at]cii.fc.ul.pt) ; 29-5 $\longrightarrow$ 2-6.

PROGRAM

Monday 30-5-2011 salle de conférences de l'IRMA

9:30 : Tea, coffee, & little cakes

10:00 : M. ARNAUDON : *Stochastic algorithms for computing means of probability measures*

11:30 : A.B. CRUZEIRO : *Stochastic Lagrangian Navier-Stokes flows*

Free time for lunch

14:00 : E. HSU : *Differentiability of reflecting B.M. and its application to path space analysis*

15:30 : J.-C. ZAMBRINI : *Integrable stochastic dynamical systems*

17:00 : T. LÉVY : *Eigenvalues of Brownian motions on classical groups*

19:30 : Dinner at the Restaurant "À la ville de Bâle", 24 rue d'Austerlitz 67000 Strasbourg (03-88-25-53-82), offered to the participants.

Tuesday 31-5-2011 salle de séminaires de l'IRMA

9:00 : Tea, coffee, & little cakes

9:30 : F. DELARUE : *Density estimate for a random noise propagating in a chain of differential equations (lower bound)*

11:00 : S. MENOZZI : *Density estimate for a random noise propagating in a chain of differential equations (upper bound)*

Free time for lunch

14:00 : S. CALOGERO : *Trend to equilibrium for kinetic F.-P. equations on Riem. manifolds*

15:30 : R. PHILIPOWSKI : *Ricci flow, coupling of Brownian motions and Perelman's L-functional*

17:00 : K. COULIBALY : *Stochastic approach to geometric flow, and application to the Ricci flow*

19:30 : Dinner at the Restaurant "La Patrie", 1 rue des balayeurs ($\approx$ place du foin) (03-88-35-16-92), very close to IRMA ; offered to the participants.

Wednesday 1-6-2011 salle de séminaires de l'IRMA

9:00 : Tea, coffee, & little cakes

9:30 : S. FANG : *Stochastic differential equations with coefficients in Sobolev spaces*

11:00 : C. TARDIF : *A study of the dynamics of the stochastic flow of a relativistic diffusion*