

SUPERVISORY INSURANCE ACCOUNTING: MATHEMATICS FOR PROVISION – AND SOLVENCY CAPITAL – REQUIREMENTS[†]

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ABSTRACT. This paper aims at providing a mathematical foundation for the terms of the well spread supervisory rule “initial market value of assets must be at least equal to provision plus solvency capital”.

It starts with a risk-adjusted assessment — given by a set of test probabilities — of the future cash-flows coming from a company business plan and attempts to define terms of a supervisory accounting mode.

First, inspired by the idea of “representation” of obligations by “equivalent” assets, we define the supervisory provision (or “liability”) attached to existing obligations. This provision is market consistent according to the mathematical definition by Cheridito, Filipovic and Kupper and satisfies a property of equilibrium between supervision’s wish for stress testing and management’s possibility for appropriate choice of assets.

The comparison between the initial market price of assets and the supervisory provision defines “solvability” of existing obligations.

In a second step the paper defines a required solvency capital as related to the level of discrepancy between assets and obligations of a company. Solvency of a business plan is defined by requiring as initial market value an additional amount over the one needed for solvability: this is the required solvency capital. A business plan with zero required solvency capital is said to have an optimal replicating asset portfolio.

It is shown that — under a natural additional condition, that of a market prudent set of test probabilities — solvability of an obligation allows for solvency of a related business plan, by choice of the asset portfolio.

The paper emphasizes the distinction between supervisory and market oriented accounting hinted to in the CEIOPS CP 20 consultative paper.

INTRODUCTION

This paper connects ideas and literatures from academia and insurance industry on the topics of “Solvency” and the convergence of accounting standards. We claim that risk assessment via sets of probability measures, called test probabilities, is a

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good foundation for this connection since it can cover topics as diverse as “stress scenarios” and “trading-risk neutral pricing probability measures”. The restriction to the last probability measures allows for the definition of *supervisory provision*, as assessment of some obligations, while the use of the whole set of test probabilities on the *net position* i.e. the difference between the future assets and claims cash-flows, shall define *free capital*.

An equivalent definition of this provision relates to the cost of (traded) assets “covering” obligations cash-flows up to some degree accepted by the supervision. Either definition includes a first protection against underwriting risks but takes definitely no account of investment risks.

We stress that these definitions have to be distinguished from other notions of provisions, used in industry practices, in particular from *technical provision* based on a argument of transfer between companies. This is a subject of ongoing research. In the rest of the paper, we simply speak of *provision* instead of supervisory provision.

Given the business plan of an insurance company, we define the *solvability* for the couple of its initial asset market value and obligations cash-flow by requiring the former to be greater than the provision of the latter. This does not depend on the investment risk of the firm.

To obtain an assessment of all risks, both investment risk and underwriting risk, the full set of test probabilities is used (in particular stress scenarios which in general are not pricing measures!) to assess the free capital and to define *required solvency capital* as a measure of the lack of congruence between assets and obligations “centered” around the initial market value and the provision respectively.

It is shown that provision, required solvency capital and free capital add up to the initial market price of assets, which is a “supervisory accounting” equality. *Solvency* of a business plan being defined by the fact that initial market value is greater than provision plus required solvency capital, is equivalent to the positivity of free capital, i.e. to the risk measure concept of acceptability.

If we allow changes in asset composition and regard the levels of initial asset market value required by the full risk evaluation i.e. the solvency condition, we find exactly as their lower bound the provision of the obligations. This means that (asymptotically) the required solvency capital can be reduced to zero by changing the asset side of the business plan. Under the additional assumption of a market prudent set of test probabilities a convenient asset rearrangement exists. Such an “optimal replicating portfolio” – as we call it – backs indeed the given obligations in an optimal way by minimizing the acceptability level of initial asset market value to the mere provision.

To ascertain solvency of a business plan within a specific company, we are confronted by a two-level control: A first “*conditio sine qua non*” is the condition: initial asset market value greater than provision. If this requirement is not satisfied, the company has to collect new cash until this goal is reached. Once this first control overcome, there are two ways to solvency. In the first case, the company complies with the solvency condition of the business plan: initial market value of assets greater than provision plus required solvency capital. If necessary, new cash has to be invested in an “eligible asset” which seen from the supervisor’s point of view is “risk-free”, in order to stay with the chosen investment risk and thus avoiding a new solvency procedure. In the second case, the company changes its portfolio to decrease its required solvency capital until the solvency condition is

reached.

Note that we address here the case of only one period of uncertainty. This is a more severe restriction concerning risk horizon, i.e. the expected time horizon for already insured casualties to come up, than concerning regulatory compliance horizon.

1. PROVISION

1.1 Use of test probabilities.

The mathematical framework is given by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ which represents the various states of nature at the future date 1 and is assumed finite for mathematical simplicity. The elements of the space $L = L^0(\Omega, \mathcal{F})$ represent contracts (also called risky positions) entered into at date 0 and providing random cash-flows of date 1 money at date 1.

An “eligible asset” $r \in L$ is a contract available at date 0 for the price of one unit of date 0 money and carrying strictly positive cash-flows at date 1. It is chosen together with a set \mathcal{P} of probabilities on (Ω, \mathcal{F}) to define *risk-adjusted assessments*:

Definition 1.1. *Risk-adjusted assessment and acceptability.*

The set \mathcal{P} of test probabilities and the eligible asset r define for each risky position $X \in L$ of date 1 its risk-adjusted assessment of date 0

$$\Phi_{\mathcal{P},r}(X) = \inf_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}}[X/r]. \quad (1.1)$$

Acceptable risky positions are those with positive risk-adjusted assessment.

The risk-adjusted assessment $\Phi_{\mathcal{P},r}(X)$ is the negative of the risk measure $\rho(X)$ (see Artzner et al., 1999) and indicates for each position X how many units of r can be disinvested at date 0 while keeping the amended risky position $X - r \cdot \Phi_{\mathcal{P},r}(X)$ acceptable to supervision. If this value is negative, then an extra investment of $-\Phi_{\mathcal{P},r}(X)$ units of r makes the new position acceptable.

Since the transition from \mathcal{P} to its closed convex hull in the space of probability measures on (Ω, \mathcal{F}) , does not change $\Phi_{\mathcal{P},r}$, we assume \mathcal{P} closed convex. The valuation $\Phi_{\mathcal{P},r}$ is *cash-invariant* in the sense that for each $c \in \mathbb{R}$ and each $X \in L$, $\Phi_{\mathcal{P},r}(c \cdot r + X) = c + \Phi_{\mathcal{P},r}(X)$.

Remark 1. Since $\Phi_{\mathcal{P},r}(r) = 1$ is the initial price of the eligible asset, we present $\Phi_{\mathcal{P},r}$ as the supervisory subjective risk-adjusted assessment relative to the eligible asset.

Remark 2. The expression “risky position” is rather general. We shall use it for example

- (i) for an asset cash-flow $X = A_1 \geq 0$ at date 1,
- (ii) for an obligation cash-flow $X = -Z_1 \leq 0$ at date 1,
- (iii) for a net position $X = A_1 - Z_1$ at date 1.

In case (iii) we shall be interested into decomposing the possible requirement on X into a part “due to Z_1 ” (this will be the topic of provision) and a part “due to A_1 and Z_1 ” (this will be the topic of required solvency capital) when dealing with the extra initial cash $-\Phi_{\mathcal{P},r}(A_1 - Z_1)$. We moreover shall look for a canonical way to obtain such a decomposition.

Remark 3. As dealt with in Artzner, Delbaen and Koch (2009) one can easily replace the single eligible asset r by a finite set of eligible assets.

1.2 Modelling a financial market.

A financial market is given by a family of *traded* contracts: S^0, S^1, \dots, S^d . These are $d + 1$ random variables where $S^i(\omega)$ is the number of money units of date 1 provided by contract S^i in state ω . Their initial prices in date 0 money units are denoted $S_0^0, S_0^1, \dots, S_0^d$. We take S^0 as numeraire by assuming $S_0^0 = 1, S^0(\omega) > 0$ for each ω . Absence of (market) arbitrage opportunities is assumed and granted by the non-emptiness of the set $\overline{\mathcal{M}} = \{\overline{\mathbb{Q}} \mid \mathbf{E}_{\overline{\mathbb{Q}}}\left[\frac{S^i}{S^0}\right] = S_0^i \text{ for } i = 1, \dots, d\}$ of “trading-risk neutral” pricing probabilities $\overline{\mathbb{Q}}$.

We assume that the eligible asset r is *attainable* from the S^0, \dots, S^d , i.e. $r = \sum_{i=0}^d \xi_i S^i$ for some $\xi_i \in \mathbb{R}, i = 0, \dots, d$. Since we took 1 as the initial price of r , the non-arbitrage condition implies $\mathbf{E}_{\overline{\mathbb{Q}}}\left[\frac{r}{S^0}\right] = 1$ for each $\overline{\mathbb{Q}} \in \overline{\mathcal{M}}$.

The standard transformation of $\overline{\mathbb{Q}} \in \overline{\mathcal{M}}$ to $\mathbb{Q}(d\omega) = \frac{r(\omega)}{S^0(\omega)} \overline{\mathbb{Q}}(d\omega)$ gives to r the role of a numeraire such that $\mathbf{E}_{\mathbb{Q}}\left[\frac{S^i}{r}\right] = S_0^i, i = 1, \dots, d$. The set

$$\mathcal{M} = \{\mathbb{Q} \mid d\mathbb{Q} = \frac{r}{S^0} d\overline{\mathbb{Q}} \text{ for some } \overline{\mathbb{Q}} \in \overline{\mathcal{M}}\} \quad (1.2)$$

is now the set of “trading-risk neutral” pricing probabilities with respect to r . We shall denote by π the pricing functional.

It is easy to check that the set $N_{\mathcal{M}}$ of zero-cost portfolios of the S^0, \dots, S^d is equal to the set $\{D \in L \mid \mathbf{E}_{\mathbb{Q}}[D/r] = 0 \text{ for all } \mathbb{Q} \in \mathcal{M}\}$. In the following we allow ourselves to use the word portfolio both for a combination of traded assets and for the resulting value of such an asset combination.

1.3 Comparing the roles of testing and pricing probability measures.

It is important to relate risk-adjusted assessments and market values. It would indeed be dangerous to find a zero-cost portfolio of the traded contracts S^0, \dots, S^d with a strictly positive risk-adjusted assessment: any position, as risky as it might be, could be modified to one with a strictly positive risk-adjusted assessment by adding some zero-cost portfolio and using the super-additivity property.

It has been shown in Artzner et al. (1999), Section 4.3 and Delbaen (2000), Chapter 7, Theorem 15, that this inconvenient situation is avoided if and only if $\mathcal{P} \cap \mathcal{M} \neq \emptyset$. We henceforth assume this condition to hold:

Assumption $NSA(\mathcal{P}, \mathcal{M})$. *No supervisory arbitrage of the assessment \mathcal{P} via the market \mathcal{M} .*

There exist probabilities on (Ω, \mathcal{F}) which are both trading-risk neutral and testing probabilities, i.e. $\mathcal{P} \cap \mathcal{M} \neq \emptyset$.

Under $NSA(\mathcal{P}, \mathcal{M})$ it becomes natural to use the theory of convolution of risk measures providing Proposition 2.1 which compares the effect of relaxing the set of test probabilities to the effect of rebalancing via the marketed contracts.

1.4 The business plan and the trading-risk exposure of an insurance company.

A company business plan is a couple (A_1, Z_1) of a future asset cash-flow A_1 and of an obligation cash-flow Z_1 with A_1 attainable, i.e. a linear combination of the S^i , and Z_1 being the obligation resulting from the policies written at date 0. We

call $C_1 = A_1 - Z_1$ the company's *net position* at date 1. The initial market price of the asset portfolio with future value A_1 is $A_0 := \mathbf{E}_{\mathbb{Q}} [A_1/r]$ for each $\mathbb{Q} \in \mathcal{M}$.

The difference $D_1 = A_1 - r \cdot A_0$ is called the – supervisory subjective – *trading-risk exposure* of the plan as it denotes the deviation from the “safe” investment in the eligible asset r . It does not change when $c \cdot r$, $c \in \mathbb{R}$, is added to A_1 , a “translation invariance” property.

1.5 Definition of provision (“liability”) attached to some obligation Z_1 .

Before we investigate the risk-adjusted assessment of the firm's actual net position, we first regard the (minimum) cost of “covering” the obligations by a possible traded position, at the supervision's satisfaction, i.e. with positivity of the risk-adjusted assessment of the final net position (see Code des Assurances, Article R^* 332 – 1 and Committee of European Insurance and Occupational Pensions Supervisors (2007), 3.75).

Definition 1.2. *Provision related to some obligation cash-flow.*

Given the risk-adjusted assessment $\Phi_{\mathcal{P},r}$ defined by the convex set \mathcal{P} of test probabilities and the eligible asset r , the provision pertaining to some obligation cash-flow Z_1 of date 1 is defined as

$$L_0(Z_1) := \inf \{c, \exists \text{ a traded asset position } A_1, \pi(A_1) \leq c, \Phi_{\mathcal{P},r}(A_1 - Z_1) \geq 0\}. \quad (1.3)$$

This definition of provision depends on market conditions via the space of traded portfolio values. In Section 2, a representation of $L_0(Z_1)$ will be given which depends on the set \mathcal{M} of pricing measures. It will turn out that in fact provision is a saddle-point equilibrium between the management's possibility to modify the trading risk via zero-cost portfolios on one hand and the supervisor's evaluation of the possible net values via the test probabilities in \mathcal{P} on the other hand.

In a first step we provide the following writing of the “min-max” aspect of provision:

Proposition 1.1.

$$L_0(Z_1) = \inf_{D \in N_{\mathcal{M}}} \sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r]. \quad (1.4)$$

Proof. First let $A_1 = r \cdot \pi(A_1) - D$ with $D \in N_{\mathcal{M}}$ be a portfolio value with $\Phi_{\mathcal{P},r}(A_1 - Z_1) \geq 0$. Then $\Phi_{\mathcal{P},r}(-D - Z_1) \geq -\pi(A_1)$ or

$$\sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r] = -\Phi_{\mathcal{P},r}(-D - Z_1) \leq \pi(A_1)$$

and therefore

$$\inf_{D \in N_{\mathcal{M}}} \sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r] \leq L_0(Z_1).$$

Conversely, for any $D \in N_{\mathcal{M}}$ and $A_1 = -r\Phi_{\mathcal{P},r}(-D - Z_1) - D$ we have

$$\Phi_{\mathcal{P},r}(A_1 - Z_1) = \Phi_{\mathcal{P},r}(-D - Z_1) - \Phi_{\mathcal{P},r}(-D - Z_1) = 0.$$

Therefore $L_0(Z_1) \leq -\Phi_{\mathcal{P},r}(-D - Z_1) = \sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r]$ or

$$L_0(Z_1) \leq \inf_{D \in N_{\mathcal{M}}} \sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r].$$

This proves (1.4). □

1.6 The solvability requirement on a couple (A_0, Z_1) .

It will turn out in Corollary 5.1 that the following relation between provision and initial market value of assets is a minimal supervisory requirement:

Definition 1.3. *Supervisory notion of “being solvable”.*

Given a supervisory assessment $\Phi_{\mathcal{P},r}$, the couple (A_0, Z_1) is solvable if A_0 , the market price of the assets at date 0, is at least as large as the provision $L_0(Z_1)$ pertaining to the obligation Z_1 :

$$A_0 \geq L_0(Z_1). \quad (1.5)$$

Remark 1. Given a business plan (A_1, Z_1) , solvability of the deduced couple (A_0, Z_1) depends on the initial market prices of the (tradeable) assets via their total value A_0 , but not on the trading-risk exposure $D_1 = A_1 - r \cdot A_0$ i.e. solvability depends on the asset portfolio through its initial market value but not through its composition: given an obligation Z_1 a portfolio A_1 of government bonds and an initially equally market valued portfolio A'_1 of “dot com” shares do simultaneously provide solvability, or not. Therefore a complete supervisory opinion on a business plan (A_1, Z_1) should depend on more than on a solvability statement on (A_0, Z_1) . This aspect will be treated in Sections 4 and 5.

Remark 2. In our single period model, the case of participatory policies does not contradict the independence of L_0 and A_1 stated in Remark 1: if any participatory payment is due at date 1, money for it is, by definition, available. Therefore it is not a topic for supervision at date 0. The situation will be different in a two period model where a profit at date 1 due for participatory payment either at date 1 or date 2 raises at date 1 a question of the sufficiency of company’s wealth at date 2 to ensure the date 2 claim payment. This shall be a matter of solvability at date 1 hence also at date 0.

2. MARKET CONSISTENT REPRESENTATION OF PROVISION

2.1 Market consistent assessment.

We shall compare assessment via \mathcal{P} to assessment via the restricted set $\mathcal{P} \cap \mathcal{M}$ of test probabilities. The later will have the “market consistency property” as it has been defined by Cheridito, Filipovic and Kupper (2008):

Definition 2.1. *Market consistent assessment.*

A mapping Φ of the set L of contracts into \mathbb{R} is market consistent if for each contract $X \in L$ and each traded contract U we have $\Phi(X + U) = \Phi(X) + \pi(U)$.

Remark. Market consistency appears as an extension of the cash invariance property $\Phi(X + c \cdot r) = \Phi(X) + c$.

It has been stated by Cheridito, Filipovic and Kupper (2008) that a mapping of the form $\Phi_{\mathcal{P}'}$ is market consistent if and only if the test probabilities $\mathbb{Q} \in \mathcal{P}'$ all belong to \mathcal{M} . In particular, $\Phi_{\mathcal{P} \cap \mathcal{M},r}$ is a market consistent assessment.

2.2 The concave convolution.

Since \mathcal{P} and \mathcal{M} are closed convex sets with $\mathcal{P} \cap \mathcal{M} \neq \emptyset$, we know (see Delbaen (2000), Section 4.3, Proposition 3 and Example 11) that $\Phi_{\mathcal{P} \cap \mathcal{M},r}$ is the (concave) convolution of $\Phi_{\mathcal{P},r}$ with $\Phi_{\mathcal{M},r}$ as defined by

$$\begin{aligned}\Phi_{\mathcal{P} \cap \mathcal{M}, r}(X) &= \Phi_{\mathcal{P}, r} \square \Phi_{\mathcal{M}, r}(X) := \sup_{Y \in L} \{ \Phi_{\mathcal{P}, r}(X - Y) + \Phi_{\mathcal{M}, r}(Y) \} \\ &= \sup \{ \Phi_{\mathcal{P}, r}(X - Y), \Phi_{\mathcal{M}, r}(Y) \geq 0 \},\end{aligned}\quad (2.1)$$

(see also Barrieu and El Karoui (2005) and Klöppel and Schweizer (2007), Section 3, Theorem 7).

In fact, one can even show the following result:

Proposition 2.1.

$$\Phi_{\mathcal{P} \cap \mathcal{M}, r}(X) = \sup \{ \Phi_{\mathcal{P}, r}(X + D), D \in N_{\mathcal{M}} \}.\quad (2.2)$$

Proof. Let $\mathcal{A}_{\mathcal{M}, r} = \{Y \in L, \Phi_{\mathcal{M}, r}(Y) \geq 0\}$. We want to show that

$$\mathcal{A}_{\mathcal{M}, r} = L_+ + N_{\mathcal{M}}.$$

The relation $\mathcal{A}_{\mathcal{M}, r} \supset L_+ + N_{\mathcal{M}}$ is obvious. For the converse inclusion, we first note that $L_+ \cap N_{\mathcal{M}} = \{0\}$, since there exists $\mathbb{Q} \in \mathcal{M}$, equivalent to \mathbb{P} (Föllmer and Schied (2004), Theorem 1.54). This implies that $L_+ + N_{\mathcal{M}} \subset \mathbb{R}^\Omega$ is closed (Föllmer and Schied (2004), Proposition 1.67). Assume for some $X \in L$ that $X \notin L_+ + N_{\mathcal{M}}$. By the separation theorem, we find — after normalization — a probability \mathbb{Q} such that $\mathbf{E}_{\mathbb{Q}}(X/r) < 0 \leq \mathbf{E}_{\mathbb{Q}}(Y/r)$ for all $Y \in L_+ + N_{\mathcal{M}}$. In particular, $\mathbf{E}_{\mathbb{Q}}(Y) = 0$ for all $Y \in N_{\mathcal{M}}$, such that $\mathbb{Q} \in \mathcal{M}$ and $X \notin \mathcal{A}_{\mathcal{M}, r}$. This proves the stated equality.

We get $\Phi_{\mathcal{P} \cap \mathcal{M}, r}(X) = \sup \{ \Phi_{\mathcal{P}, r}(X - Y), Y \in L_+ + N_{\mathcal{M}} \} = \sup \{ \Phi_{\mathcal{P}, r}(X + D), D \in N_{\mathcal{M}} \}$. \square

Remark. Proposition 2.1 can be interpreted as linking the search for improving risk-adjusted assessment $\Phi_{\mathcal{P}, r}$ — with the help of zero-cost market transactions — to the market consistent evaluation $\Phi_{\mathcal{P} \cap \mathcal{M}, r}$.

2.3 Market consistency and saddle-point representation of provision.

As a functional of the obligation cash flow the provision is market consistent:

Proposition 2.2.

$$L_0(Z_1) = -\Phi_{\mathcal{P} \cap \mathcal{M}, r}(-Z_1) = \sup_{\mathbb{Q} \in \mathcal{P} \cap \mathcal{M}} \mathbf{E}_{\mathbb{Q}}[Z_1/r].\quad (2.3)$$

Proof. By Propositions 1.4 and 2.1, we have

$$\begin{aligned}L_0(Z_1) &= \inf_{D \in N_{\mathcal{M}}} \sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}}[(D + Z_1)/r] = - \sup_{D \in N_{\mathcal{M}}} \inf_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}}[-(D + Z_1)/r] \\ &= - \sup_{D \in N_{\mathcal{M}}} \Phi_{\mathcal{P}, r}(-D - Z_1) = -\Phi_{\mathcal{P} \cap \mathcal{M}, r}(-Z_1) = \sup_{\mathbb{Q} \in \mathcal{P} \cap \mathcal{M}} \mathbf{E}_{\mathbb{Q}}[Z_1/r].\end{aligned}$$

\square

Remark. We notice that

- (i) (2.3) depends on market conditions via the set \mathcal{M} of pricing measures (as mentioned before),
- (ii) if \mathcal{M} is a singleton (complete markets) then the provision equals the price of the obligation,
- (iii) if $\mathcal{P} \supset \mathcal{M}$ (as in the Appendix example of Artzner, Delbaen and Koch (2009)), then $L_0(Z_1)$ becomes $\sup_{\mathbb{Q} \in \mathcal{M}} \mathbf{E}_{\mathbb{Q}}[Z_1/r]$ the actual superhedging price of the risky position Z_1 (see Föllmer and Schied (2004), Remark 1.32).

The set \mathcal{P} contains the set of test probabilities which from the supervisor's point of view are necessary for a prudent estimation of the underwriting risk. Therefore, the intersection $\mathcal{P} \cap \mathcal{M}$ in (2.3) is a natural compromise between being close to market and sufficiently prudent. This aspect will become even more evident in the following proposition:

Proposition 2.3.

$$\begin{aligned} L_0(Z_1) &= \inf_{D \in N_{\mathcal{M}}} \sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r] \\ &= \sup_{\mathbb{Q} \in \mathcal{P}} \inf_{D \in N_{\mathcal{M}}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r]. \end{aligned} \quad (2.4)$$

Proof. By Propositions 1.1 and 2.2, we have

$$\begin{aligned} L_0(Z_1) &= \inf_{D \in N_{\mathcal{M}}} \sup_{\mathbb{Q} \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r] \geq \sup_{\mathbb{Q} \in \mathcal{P}} \inf_{D \in N_{\mathcal{M}}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r] \\ &\geq \sup_{\mathbb{Q} \in \mathcal{P} \cap \mathcal{M}} \inf_{D \in N_{\mathcal{M}}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r] = \sup_{\mathbb{Q} \in \mathcal{P} \cap \mathcal{M}} \mathbf{E}_{\mathbb{Q}} [Z_1/r] = L_0(Z_1) \end{aligned}$$

which gives (2.4). \square

Proposition 2.3 shows that provision is an equilibrium between the supervisor's intention to maximize the risk evaluation of Z_1 with respect to the set \mathcal{P} of test probabilities and the company's possibility to minimize it by an appropriate zero-cost portfolio. We emphasize however that in general the company's objective is not to minimize the cost of the supervisor's requirement.

3. OPTIMAL REPLICATING PORTFOLIO

3.1 A counter example.

In general, unfortunately, we cannot expect that the "inf" in equality (1.4) of Proposition 1.1 (equivalent to "sup" in (2.2)) is attained, as the following example will show:

Example. Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$. In the following, we write probabilities and random variables as row vectors in \mathbb{R}^3 . Thus, let $\mathbb{P} = (1/3, 1/3, 1/3)$, $S^0 = r \equiv 1$, $S^1 = (0, 3, 0)$ with $S_0^1 = 1$, ($d = 1$), such that the set of trading-risk neutral probabilities is $\mathcal{M} = \{(\mu, 1/3, 2/3 - \mu), \text{ for } 0 \leq \mu \leq 2/3\}$, $\mathbb{P} \in \mathcal{M}$, and $N_{\mathcal{M}} = \mathbb{R} \cdot V$ with $V = (-1, 2, -1)$.

We set $h(x) := 1 - 4x + 6x^2$ and define

$$\mathcal{P} := \{(q_1, q_2, 1 - q_1 - q_2), \text{ with } 0 \leq q_1 \leq 1/2, h(q_1) \leq q_2 \leq 1 - q_1\}.$$

\mathcal{P} is a convex set in \mathbb{R}^3 whose set of exposed points is $\mathcal{P}^{ex} = \{\mathbb{Q}_q = (q, h(q), 1 - q - h(q)), 0 \leq q \leq 1/2\}$. Since $h(x) \geq 1/3$ and $h(x) = 1/3$ if and only if $x = 1/3$, we find $\mathcal{P} \cap \mathcal{M} = \{\mathbb{P}\}$.

For $X = (1, 0, -1)$, we find $\Phi_{\mathcal{P} \cap \mathcal{M}, r}(X) = \mathbf{E}_{\mathbb{P}}[X] = 0$. In order to calculate $\sup \{\Phi_{\mathcal{P}, r}(X + D), D \in N_{\mathcal{M}}\} = \sup_{\alpha \in \mathbb{R}} \{\inf_{0 \leq q \leq 1/2} \mathbf{E}_{\mathbb{Q}_q}[X + \alpha V]\}$, we first note that $\mathbf{E}_{\mathbb{Q}_q}[X + \alpha V] = 2q + h(q) - 1 + \alpha(3h(q) - 1) = 2\alpha - (2 + 12\alpha)q + (6 + 18\alpha)q^2$. The minimum is attained for $q^*(\alpha) = \mathbf{1}_{[-1/6, +\infty)}(\alpha) \frac{2+12\alpha}{12+36\alpha}$ such that

$g(\alpha) := \inf_{0 \leq q \leq 1/2} \mathbf{E}_{\mathbb{Q}_q} [X + \alpha V] = \mathbf{1}_{(-\infty, -1/6)}(\alpha) 2\alpha - \mathbf{1}_{[-1/6, +\infty)}(\alpha) \frac{1}{6+18\alpha} < 0$. The function g has its supremum ($= 0$) only in the limit $\alpha \rightarrow \infty$.

Note that $\{\mathbf{E}_{\mathbb{Q}} [V], \mathbb{Q} \in \mathcal{P}\} = \{2 - 12q + 18q^2, 0 \leq q \leq 1/2\} = [0, 2]$ and therefore $V \in \mathcal{A}_{\mathcal{P},r} \cap N_{\mathcal{M}}$: the sufficient condition in Föllmer and Schied (2004) Proposition 1.67 for the closure of $\mathcal{A}_{\mathcal{P},r} + N_{\mathcal{M}}$, is not satisfied. Here, it is also not sufficient: indeed with $X_n = X + \frac{1}{6+18n}r + nV$ and $D_n = -nV$, we get a sequence $Y_n = X_n + D_n = X + \frac{1}{6+18n}r \in \mathcal{A}_{\mathcal{P},r} + N_{\mathcal{M}}$, converging to $X \notin \mathcal{A}_{\mathcal{P},r} + N_{\mathcal{M}}$, since $X + \alpha V \notin \mathcal{A}_{\mathcal{P},r}$ for all $\alpha \in \mathbb{R}$.

3.2 Optimal trading-risk exposure.

Taking into account Proposition 2.1 and the former counter example we focus on the existence of zero-cost rebalancing with optimal full risk assessment:

Definition 3.1. *Optimal trading-risk exposure.*

Given a risk assessment $\Phi_{\mathcal{P},r}$, a zero cost portfolio D_1^* is called an optimal trading-risk exposure for a contract $X \in L$ if

$$\Phi_{\mathcal{P},r}(X + D_1^*) = \Phi_{\mathcal{P} \cap \mathcal{M},r}(X). \quad (3.1)$$

A business plan (A_1^*, Z_1) with optimal trading-risk exposure for $-Z_1$ has its full risk assessment linked to the mere solvability of (A_0, Z_1) :

Proposition 3.1. *Optimal replicating portfolio.*

- (i) An asset portfolio A_1^* has a optimal trading-risk exposure $D_1^* = A_1^* - r \cdot A_0$ for $-Z_1$ if and only if

$$\Phi_{\mathcal{P},r}(A_1^* - Z_1) = A_0 - L_0(Z_1). \quad (3.2)$$

Such a portfolio is called an optimal replicating portfolio for the obligation Z_1 .

- (ii) If A_1^* is an optimal replicating portfolio for Z_1 , so is $A_1^* + c \cdot r$ for any constant $c \in \mathbb{R}$ (cash invariance property).

Proof. (i) $A_1^* = r \cdot A_0 + D_1^*$ is an optimal replicating portfolio for Z_1 if and only if

$$\Phi_{\mathcal{P},r}(A_1^* - Z_1) = A_0 + \Phi_{\mathcal{P},r}(D_1^* - Z_1) = A_0 + \Phi_{\mathcal{P} \cap \mathcal{M},r}(-Z_1) = A_0 - L_0(Z_1).$$

(ii) is obvious. □

Remark. Optimal replicating portfolio has been mentioned as a portfolio which “immunizes the liability cash-flows against all changes in the underlying market risk factors” (see Federal Office of Private Insurance (2006), Appendix C).

The example in Section 3.1 shows that we need an additional assumption in order to ascertain the existence of an optimal replicating portfolio:

Definition 3.2. *A market prudent set of test probabilities.*

The set \mathcal{P} of test probabilities is called market prudent if for each non zero $D \in N_{\mathcal{M}}$ the strict inequality

$$\Phi_{\mathcal{P},r}(D) < 0 \quad (3.3)$$

holds.

Remark. The market prudence condition is the extension to all traded assets of the “non acceptability of leverage” condition $NAL(\mathcal{A})$ in Section 1 of Artzner, Delbaen and Koch (2009). It allows to “detect” leveraged (zero-cost) portfolios as unacceptable.

Obviously, a market prudent set of test probabilities satisfies $\mathcal{P} \cap \mathcal{M} \neq \emptyset$. In the example above we had $\Phi_{\mathcal{P},r}(V) = 0$. Therefore this \mathcal{P} is not market prudent.

Note that \mathcal{P} is market prudent if and only if for any attainable future asset value A_1 with market price A_0 and different from $r \cdot A_0$ we have

$$\Phi_{\mathcal{P},r}(A_1) < A_0.$$

Or to put it differently, for a market prudent \mathcal{P} multiples $r \cdot A_0$ of the eligible asset are the only attainable assets assessed via $\Phi_{\mathcal{P},r}$ by their date 0 market price A_0 .

Proposition 3.2.

Let \mathcal{P} be a market prudent set of test probabilities. Then for each $X \in L$, there exists $D^* \in N_{\mathcal{M}}$ such that

$$\Phi_{\mathcal{P} \cap \mathcal{M},r}(X) = \Phi_{\mathcal{P},r}(X + D^*). \quad (3.4)$$

Proof. Let $X \in L$ and $D_n \in N_{\mathcal{M}}$ with $\Phi_{\mathcal{P},r}(X + D_n) \nearrow \Phi_{\mathcal{P} \cap \mathcal{M},r}(X)$. If $\|D_n\|$ stays bounded, then – for a subsequence – $D_n \rightarrow D^* \in N_{\mathcal{M}}$ and we find that $\Phi_{\mathcal{P},r}(X + D^*) = \Phi_{\mathcal{P} \cap \mathcal{M},r}(X)$. If otherwise $\|D_n\| \nearrow +\infty$, then – again for a subsequence – $D_n/\|D_n\| \rightarrow D \in N_{\mathcal{M}}$ with $D \neq 0$. Now, $\Phi_{\mathcal{P},r}(D) = \lim_n \frac{1}{\|D_n\|} \Phi_{\mathcal{P},r}(X + D_n) = 0$. However, the market prudence condition requires $\Phi_{\mathcal{P},r}(D) < 0$ for all $D \in N_{\mathcal{M}}, D \neq 0$, a contradiction. \square

Obviously, market prudence implies the existence of an optimal replicating portfolio:

Corollary 3.1.

Assume \mathcal{P} to be market prudent. Then for any (A_0, Z_1) there exists an optimal replicating portfolio A_1^* , obtained by rebalancing the given portfolio.

Since in our case $\emptyset \neq \mathcal{P} \cap \mathcal{M}$ is compact, market prudence implies the existence of a saddle-point:

Corollary 3.2.

If \mathcal{P} is market prudent, then for all Z_1 there exists a saddle-point $(D^*, \mathbb{Q}^*) \in N_{\mathcal{M}} \times (\mathcal{P} \cap \mathcal{M})$ such that

$$\mathbf{E}_{\mathbb{Q}}[(D^* + Z_1)/r] \leq \mathbf{E}_{\mathbb{Q}^*}[Z_1/r] = L_0(Z_1) = \mathbf{E}_{\mathbb{Q}^*}[(D + Z_1)/r] \quad (3.5)$$

for all $D \in N_{\mathcal{M}}$ and $\mathbb{Q} \in \mathcal{P}$.

Proof. \mathcal{P} being market prudent, we find $D^* \in N_{\mathcal{M}}$ with

$$L_0(Z_1) = \sup_{\mathbb{Q}' \in \mathcal{P}} \mathbf{E}_{\mathbb{Q}'}[(D^* + Z_1)/r] \geq \mathbf{E}_{\mathbb{Q}}[(D^* + Z_1)/r] \quad (3.6)$$

for all $\mathbb{Q} \in \mathcal{P}$. If $\mathbb{Q} \in \mathcal{P} \setminus \mathcal{M}$, then $\mathbf{E}_{\mathbb{Q}}(D/r) \neq 0$ for some $D \in N_{\mathcal{M}}$ such that $\inf_{D \in N_{\mathcal{M}}} \mathbf{E}_{\mathbb{Q}}[(D + Z_1)/r] = -\infty$. The compactness of $\mathcal{P} \cap \mathcal{M} \neq \emptyset$ implies the

existence of $\mathbb{Q}^* \in \mathcal{P} \cap \mathcal{M}$ with $\mathbf{E}_{\mathbb{Q}^*} [Z_1/r] \geq \mathbf{E}_{\mathbb{Q}} [Z_1/r] = \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r]$ for all $\mathbb{Q} \in \mathcal{P} \cap \mathcal{M}$ and $D \in N_{\mathcal{M}}$. This implies

$$\mathbf{E}_{\mathbb{Q}^*} [(D + Z_1)/r] = \mathbf{E}_{\mathbb{Q}^*} [Z_1/r] = \sup_{\mathbb{Q} \in \mathcal{P}} \inf_{D \in N_{\mathcal{M}}} \mathbf{E}_{\mathbb{Q}} [(D + Z_1)/r] = L_0(Z_1) \quad (3.7)$$

for all $D \in N_{\mathcal{M}}$. (3.6) and (3.7) prove (3.5). \square

4. REQUIRED SOLVENCY CAPITAL

4.1 Definition of the required solvency capital.

The expressions $A_1 - r \cdot A_0$ and $Z_1 - r \cdot L_0(Z_1)$ are the deviations of assets and obligations from their “reference points” given by market trading and provisioning, respectively. The difference of these deviations is $A_1 - r \cdot A_0 - (Z_1 - r \cdot L_0(Z_1)) = C_1 - r \cdot \Phi_{\mathcal{P} \cap \mathcal{M}, r}(C_1)$ a kind of centered net position for the business plan. Risk assessment of “asset-liability management” evaluates this difference and is a measure of the inadequacy between assets and obligations.

It will be used by supervision to constrain the excess of market value of assets on top of the provision to be larger than a minimum called required solvency capital:

Definition 4.1. *Required solvency capital.*

Given the risk-adjusted assessment $\Phi_{\mathcal{P}, r}$ and the business plan (A_1, Z_1) with net position $C_1 = A_1 - Z_1$, the required solvency capital $M_0(C_1)$ is given by

$$\begin{aligned} M_0(C_1) &:= -\Phi_{\mathcal{P}, r}(A_1 - r \cdot A_0 - (Z_1 - r \cdot L_0(Z_1))) \\ &= -\Phi_{\mathcal{P}, r}(C_1 - r \cdot \Phi_{\mathcal{P} \cap \mathcal{M}, r}(C_1)). \end{aligned} \quad (4.1)$$

Remark 1. Since in this paper we treat only supervisory provision (in contrast to technical provision), the definition of required solvency capital incorporate only *one* part of what is called in insurance industry *solvency capital requirement* (SCR). It is the part which depends heavily on the company asset portfolio and is equal to zero for the optimal replicating portfolio (Corollary 4.1). For the second part of the SCR, depending only on the obligations, we refer to Eisele and Artzner (2010).

Remark 2. For constants c, d , the business plans (A_1, Z_1) and $(A_1 + c \cdot r, Z_1 + d \cdot r)$ have the same required solvency capital. Since $c \cdot r$ and $d \cdot r$ are considered as “safe” (see Section 1.4), this invariance is in line with the description of required capital as “a level of capital that enables an insurance undertaking to absorb significant unforeseen losses” (see Committee of European Insurance and Occupational Pensions Supervisors (2007), 2.21).

The following property is another sign of the relevance of the required solvency capital:

Proposition 4.1.

The required solvency capital is non-negative:

$$M_0(C_1) = -\Phi_{\mathcal{P}, r}(C_1) + \Phi_{\mathcal{P} \cap \mathcal{M}, r}(C_1) \geq 0. \quad (4.2)$$

The inequality is obvious.

Corollary 4.1.

An asset portfolio $A_1^* = rA_0 + D_1^*$ is an optimal replicating portfolio for an obligation Z_1 if and only if the required solvency capital $M_0(C_1^*)$ of the net position $C_1^* = A_1^* - Z_1$ is zero:

$$M_0(C_1^*) = 0. \quad (4.3)$$

Proof. Since $\Phi_{\mathcal{P} \cap \mathcal{M}, r}(C_1^*) = A_0 - L_0(Z_1)$, (4.3) is equivalent to (3.2). \square

4.2 Free capital and the supervisory accounting equality.

Definition 4.2. *Free capital of a business plan.*

Given the risk-adjusted assessment $\Phi_{\mathcal{P}, r}$ and the business plan (A_1, Z_1) with net position $C_1 = A_1 - Z_1$, we call free capital the number

$$F_0(C_1) := \Phi_{\mathcal{P}, r}(C_1). \quad (4.4)$$

Proposition 4.2. *Supervisory accounting equality of a business plan.*

For the business plan (A_1, Z_1) with net position $C_1 = A_1 - Z_1$ the following “supervisory accounting equality” holds:

$$\pi(A_1) = A_0 = L_0(Z_1) + M_0(C_1) + F_0(C_1). \quad (4.5)$$

Proof. We have by (4.4), (4.2) and cash invariance

$$\begin{aligned} F_0(C_1) &= \Phi_{\mathcal{P}, r}(C_1) \\ &= \Phi_{\mathcal{P} \cap \mathcal{M}, r}(C_1) + \Phi_{\mathcal{P}, r}(C_1 - r \cdot \Phi_{\mathcal{P} \cap \mathcal{M}, r}(C_1)) \\ &= A_0 - L_0(Z_1) - M_0(C_1). \end{aligned}$$

\square

5. SOLVENCY CONDITION AND FREE CAPITAL

5.1 Solvency condition of a business plan (A_1, Z_1) .

It is now very natural to compare initial market value to the total of (required) provision and required solvency capital:

Definition 5.1. *Solvency condition of a business plan.*

The insurance company’s business plan (A_1, Z_1) satisfies the solvency condition if and only if the initial market price A_0 of its assets is greater than provision plus required solvency capital:

$$\pi(A_1) = A_0 \geq L_0(Z_1) + M_0(A_1 - Z_1). \quad (5.1)$$

If \mathcal{P} is market prudent, then a “solvable” insurance company can always satisfy the solvency condition by rebalancing its portfolio:

Corollary 5.1.

Let \mathcal{P} be market prudent. If (A_0, Z_1) is solvable, then the business plan (A_1^*, Z_1) with an optimal replicating portfolio A_1^* and $A_0 = \pi(A_1^*)$ satisfies the solvency condition.

Proof. This follows from Definition 1.3 and Corollary 4.1. \square

The cash-invariance property of $\Phi_{\mathcal{P},r}$ ensures that (A_1, Z_1) satisfies the solvency condition if (and only if) $A_0 = \pi(A_1) \geq -\Phi_{\mathcal{P},r}(D_1 - Z_1) = L_0(Z_1) + M_0(A_1 - Z_1)$. We call $-\Phi_{\mathcal{P},r}(D_1 - Z_1)$ the *solvency level* for the initial asset market value A_0 , due to the obligation Z_1 coupled with the trading-risk exposure D_1 .

If a company with obligation Z_1 and a projected trading-risk exposure D_1 has an initial asset market value $A_0 < -\Phi_{\mathcal{P},r}(D_1 - Z_1)$ and if it does not want to change its exposure it has

- (i) to get an additional amount of cash $c \geq -\Phi_{\mathcal{P},r}(D_1 - Z_1) - A_0$ from the capital market,
- (ii) to invest c in the eligible asset r ,

since D_1 is then unchanged and $\Phi_{\mathcal{P},r}(A_1 + c \cdot r - Z_1) = A_0 + c + \Phi_{\mathcal{P},r}(D_1 - Z_1) \geq 0$.

The company thereby avoids a new assessment of the final future net position and, possibly, a new requirement! The shareholders will receive at the final date 1 the amount $\max(A_1 + c \cdot r - Z_1, 0)$.

5.2 Risk-adjusted assessment again.

The supervisory accounting equality brings us back to our starting point, the theory of risk measurement, equivalently risk-adjusted assessments:

Proposition 5.1.

The insurance company's business plan (A_1, Z_1) satisfies the solvency condition if and only if its free capital $F_0(A_1 - Z_1) = \Phi_{\mathcal{P},r}(A_1 - Z_1)$ is non-negative.

6. SUPERVISORY INSURANCE ACCOUNTING

At this point we can have a “bird’s-eye view” of what “supervisory accounting” could be. Given a future net cash-flow $C_1 = A_1 - Z_1$ the risk measure type element $\Phi_{\mathcal{P},r}(C_1)$ would be the main supervisory item, also called “free capital” $F_0(C_1)$. The company is totally free to dispense with its free capital, as long as it becomes non-negative.

At the opposite, a mere definition concerns the obligation Z_1 of the company, that of $L_0(Z_1)$ the “supervisory provision”. Provision reflects three topics: covering of obligations, market consistent assessment of obligations and equilibrium between scenario testing (by supervisors) and asset choice (by management).

From comparison between provision and initial market value of assets is derived the “solvability” property which rather remarkably can ensure the non-negativity of free capital (or equivalently solvency) after some asset rearrangement.

As mentioned in the introduction we do not treat here the concept of technical provision, necessary for a clear distinction between the accounting items described by the German words “Eigenkapital” versus “Fremdkapital”. We refer to Eisele and Artzner (2010) for a more detailed analysis.

One attempt of the paper has been to give a characterization of the decomposition $A_0 - F_0 = L_0 + M_0$ of the supervisory term $A_0 - F_0$.

CONCLUSION

We have shown that the mathematical formalism of risk measures or risk-adjusted assessment bears relevance to industry practices, for example with respect to the two step procedure:

- (i) dealing first with underwriting risk per se, albeit in a market environment,
- (ii) dealing with asset-liability mismatch risk.

The two *definitions* of “required” provision and required solvency capital as well as the *statement* of solvency, all originate in a general approach to risk *measurement* of an intended net position. Some risk *management* leading to an optimal replicating portfolio is also amenable to the formalism.

We hope to have contributed material for the ongoing discussions on distinctions between market-oriented and supervision-oriented accountings.

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