Geometry, dynamics and physics: 100th Encounter between Mathematicians and Theoretical Physicists
Institut de Recherche Mathématique Avancée (University of Strasbourg and CNRS)
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Organisation: Norbert A'Campo, Athanase Papadopoulos, and Sumio Yamada

Titles and abstracts of talks

1. Daniel Bennequin (Univ. Paris, Jussieu): A geometric explanation of the standard model structure

   Summary: We prove that the spontaneous gauge symmetry breacking of a generic gravity and spinor dynamics in twelve lorentzian dimensions along a four-dimensional submanifold, leads to the full content of known particles (several generations of quarks and leptons, electro-weak and strong interactions bosons), plus other effects that are going beyond the standard model. The main mechanism underlying this phenomenon is triality in eight euclidian dimensions. (Ref: Arxiv General Physics 1601.04127, LN graduate course Geometry of Particles, P7 2014-2016, available on demand.)

2. Klaus Schmidt (ESI, Vienna): Entropy and periodic points of algebraic actions of discrete groups

   Abstract: Let G be a countable group of automorphisms of a compact abelian group X. A point $x \in X$ is G-periodic if its G-orbit is finite. This talk addresses the following problems:

   1. When does G have nontrivial periodic points?

   2. When are the G-periodic points dense in X?

   3. When is the logarithmic growth rate of the number of G-periodic points equal to the topological entropy of the G-action on X?

Recent progress on these questions is due to S. Bhattacharya (for the problems 1. and 2.) and V. Dimitrov (problem 3.).

3. Wendelin Werner (ETH, Zürich): Conformal loop ensembles on Liouville quantum gravity

   Abstract: We will explain how explorations of conformal loop ensembles (and their loop-trunk decompositions of these explorations) on some independent random "quantum surfaces" give rise to (and can be encoded by) simple growth/fragmentation processes. This description has implications for the study of Liouville quantum gravity in the case of general ?, and yields also some new results on SLE itself. This is based on ongoing joint work with Jason Miller and Scott Sheffield.

4. Marc Rosso (Univ. Paris, Jussieu): TBA

5. Jürgen Jost (MPI Leipzig): From harmonic mappings to the nonlinear supersymmetric sigma model. A challenge from QFT for Geometric Analysis.

   Abstract: The nonlinear supersymmetric sigma model of QFT couples a harmonic-like map from a Riemann surface with its superpartner, an anticommuting spinor field along that map. The conformal structure of the Riemann surface then also becomes a variable, and it has a superpartner, the gravitino field. We develop the corresponding mathematical theory, both in the original super setting and in a version that involves only commuting variables. The latter leads to challenging problems in nonlinear partial differential equations, and it requires the most advanced techniques of that field.

6. Catherine Meusburger (Univ. Erlangen): Ideal tetrahedra and their duals in 3d gravity

   Abstract : We use a unified description of 3d hyperbolic space, anti de Sitter space and so called half-pipe space to obtain a unified description of ideal tetrahedra in these spaces and a unified formula for their volumes.

   We also incorporate 3d de Sitter, Minkowski, anti de Sitter space in this picture, describe the duality between these spaces and the first three and discuss dual ideal tetrahedra in these spaces.

   This is ongoing work with Carlos Scarinci.

7. Gabriele Mondello (Univ. Rome 1): On the existence of spherical metrics with conical points

   Abstract: For every compact connected surface with negative Euler characteristic Koebe and Poincaré proved the existence and uniqueness of a hyperbolic metric in each conformal class. McOwen and Troyanov proved the same result for metrics with prescribed conical behavior at given marked points. The analogous question for metrics of curvature 1 is subtler: in a small range existence and uniqueness still holds (Troyanov), in another range existence holds but uniqueness fails (Bartolucci-De Marchis-Malchiodi). The aim of this talk is to discuss some results of existence and non-existence of metrics of curvature 1 on surfaces with conical singularities. This is joint work with Dmitri Panov.

8. Basilis Gidas (Brown Univ.): Towards a Probabilistic/Combinatorics Framework for Cognition and Biology

   Abstract: The impressive success of physics in the past century (centuries!) is, in a sense, a success of mathematics. This will be more so – practitioners in the area believe – for the story of biology/cognition and mathematics in the future. The reason for this is that biology is, in a way, physics plus evolution. It is physics that controls the interactions among the components of a biological system. But it is evolution that produced – over four billion years - the perfect "code" or perfect "syntactic language" for the folding of proteins, the collective behavior of the assembly of genes in Gene Regulatory Networks, the collective behavior of proteins (and lipids and a host of smaller molecules) participating in Signal Transduction Pathways associated with cell-growth, cell division, cell differentiation, apoptosis, and immunology. It is evolution that governs the language of complex cellular networks in the brain, and hence it is evolution that governs high-level cognition tasks such as speech and vision recognition. A complete understanding of these tasks entails mathematical models that articulate complex physical interactions and phenomena. But they must also integrate global regularities and constraints produced by evolution. Although we know many patterns associated with evolution (and we can device procedures for incorporating them), we lack an understanding of what the overall principles or "laws" of evolution are and what mathematical structures are appropriate for formulating these principles. It is believed that a combination of combinatorics, probability and topology will play a definite role towards establishing a relevant framework. In this talk, we will articulate some problems and key sources of difficulties in vision, speech and biology which make information processing difficult, and hint towards a syntactic/combinatorial framework for articulating these problems. Within this framework, we will formulate a mathematical representation of genes – a framework has been successful in finding genes in the human genome.

9. Robert C. Penner (IHES): Theory of Morphogenesis

   Abstract: A model of morphogenesis is proposed based upon seven explicit postulates. The mathematical import and biological significance of the postulates are explored and discussed. This is joint work with With A. Minarsky, N. Morozova, and C. Soulé.

10. Alberto Cattaneo (Univ. Zurich): Geometrical construction of reduced phase space

    Abstract: The reduced phase space of a field theory is the space of its possible initial conditions endowed with a natural symplectic structure. An alternative to Dirac's method, relying on natural geometric aspects of variational problems, was introduced by Kijowski and Tulczijev. This method also has the advantage of having a natural generalization in the BV context. In this talk, I will explain the method and describe some examples, focusing in particular on the tetradic version of general relativity in four dimensions.

11. Pavel Mnev (Univ. Notre Dame): Cellular BV-BFV-BF theory

    Abstract: We will present an example of a topological field theory living on cobordisms endowed with CW decomposition (this example corresponds to the so-called BF theory in its abelian and non-abelian variants), which satisfies the Batalin-Vilkovisky master equation, satisfies (a version of) Segal's gluing axiom w.r.t. concatenation of cobordisms and is compatible with cellularaggregations. In non-abelian case, the action functional of the theory is constructed out of local unimodular L-infinity algebras on cells; the partition function carries the information about the Reidemeister torsion, together with certain information pertaining to formal geometry of the moduli space of local systems. This theory provides an example of the BV-BFV programme for quantization of field theories on manifolds with boundary in cohomological formalism. This is a joint work with Alberto S. Cattaneo and Nicolai Reshetikhin.

12. Yvan Velenik (Univ. Geneva): Ornstein-Zernike theory of Ising and Potts models: a review of some applications

    Abstract: I'll review some of the results that have been obtained for Ising and Potts models on Zd, using the so-called Ornstein-Zernike theory. The latter, among other applications, enables a non-perturbative analysis of these models, away from their critical point. The first class of results that will be discussed apply for all temperature above the critical one, in any dimension: sharp asymptotics of spin-spin correlations; effect of a line of modified coupling constants on the correlation length. The second class is restricted to two-dimensional models and applies to any temperature below critical: description of typical local behavior under arbitrary boundary condition; smoothness and strict convexity of the (Wulff) equilibrium crystal shape; Brownian bridge asymptotics for the interface; interface localization by a row of weakened coupling constants; scaling limit, as phase coexistence is approached, of the interface separating a layer of unstable phase along the boundary of the system from the stable phase occupying the bulk.

13. Stanislalv Smirnov (Univ. Geneva): ntegrability in 2D lattice models, CFT ansd SLE

14. Toshikazu Sunada (Meiji Univ. Tokyo): Crystallographic Tight Frames

    Abstract: Motivated by the recent development in systematic design of crystal structures, I will discuss interesting relationships among seemingly irrelevant subjects;

say, standard crystal models, tight frames in the Euclidean space, rational points on Grassmannian, and quadratic Diophantine equations.

15. Anish Ghosh (Tata Institute Bombay): Quadratic forms and ergodic theory

    Abstract: I will discuss the Oppenheim conjecture (now a celebrated theorem of Margulis) and it's variations. In particular, I will explain some recent advances in quantitative and effective versions of the problem using the ergodic theory of group actions on homogeneous spaces.

16. Oscar Garcia Prada (ICMAT, Madrid): Kaehler-Yang-Mills equations and gravitating vortices

    Abstract: In this talk we first introduce the Kaehler-Yang-Mills equations on a holomorphic bundle over a compact complex manifold. They emerge from a natural extension of the theories for constant scalar curvature Kaehler metrics and Hermitian-Yang-Mills connections. We construct solutions to these equations by applying dimensional reduction methods to the product of the complex projective line with a compact Riemann surface. The resulting equations, that we call gravitating vortex equations, describe abelian vortices on the Riemann surface coupled to a metric (based on joint work with L. Alvarez-Consul, M. Garcia-Fernandez and V. Pingali).

17. Hubert Goenner (Univ. Goettingen): Cartan and his contributions to theoretical physics

    Abstract: A closer look is taken at Cartan's importance for theoretical physics proper. Three fields are explicated. The first is Newton-Cartan theory, a covariant formulation of Newton's theory of the gravitational field. Then, Cartan's involvement in the spinor concept for which he layed the ground, will be discussed. The third remark will be on Einstein-Cartan theory, a relativistic theory of gravitation with torsion, alternative to Einstein's theory.

18. Sorin Dumitrescu (Univ. Nice): Cartan geometries on compact manifolds.

    Abstract: This talk deals with (holomorphic) Cartan geometries on compact (complex) manifolds. One can think at the following interesting examples: affine connections, projective connections, (pseudo-) Riemannian metrics. When *flat* or *locally homogeneous*, those geometric structures are locally modeled on (complex) homogeneous spaces $X = G/H$, with G a Lie group and H a closed subgroup. We will present the deformation space of (G, X)-structures on compact manifolds and give some applications. We will also discuss classification results for compact complex manifolds bearing holomorphic Cartan geometries.